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# Transistor Amplifiers Part I and Part II

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TRANSISTOR AMPLIFIERS.

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## TRANSISTOR AMPLIFIERS PARTS I and II

-BY-

WM. MALCOLM BAUER  
Professor of Engineering Electronics  
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## TRANSISTOR AMPLIFIERS

Part I - A Simple Treatment of the Transistor as a  
Small-signal Amplifier

Part II - Transistor Parameters and Performance  
Equations

by

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//

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A SIMPLE TREATMENT OF THE TRANSISTOR  
as a  
SMALL SIGNAL AMPLIFIER

by

W. M. Bauer

October 1957

The following treatment is given as a preliminary simplified study of the transistor as an amplifier. The discussion is approximate in that internal feedback is neglected and the output impedance of the transistor is so large in comparison with the load impedance that it may be neglected. These approximations are found perfectly acceptable for low resistance loads up to something like 5000 ohms, when the parameters of a particular transistor are known by measurement. If a manufacturer's average parameters are used, this simplified treatment is all that is warranted in calculation of the performance of this transistor type in an amplifier circuit, if the load resistance is perhaps not greater than 10,000 ohms. The simplified treatment will apply to both the common base and common emitter configurations.

STATIC CHARACTERISTICS

An experimental approach to the understanding of the transistor is to obtain static characteristics. The input characteristic curve may be obtained by the circuit of figure 1. The data, when plotted, gives the input characteristic curve of figure 2. If  $V_C$  is varied, it takes a careful experimenter to see that  $V_C$  has any effect on the input characteristic. By neglecting the influence of  $V_C$ , we are disregarding





the internal feedback. The slope of the curve at the quiescent operating point gives the dynamic input resistance,  $r_i$ , which will be much larger in the common emitter configuration because the base current is much less than the emitter current.

The output characteristics may be obtained by the circuit of figure 3.

If the common emitter circuit is used, then figure 1b may be adapted to read  $I_2$  as a function of  $V_2$ , as is done in figure 3 for constant values of  $I_1$ . The set of characteristics so obtained are shown in figure 4. The reciprocal of the slope of these curves is the output resistance,  $r_{22}$ .

The feature of most importance, however, is the ratio of the increment of output current to an increment of input current, while keeping the collector voltage constant. In the case of the common base circuit, this current transfer ratio,  $\alpha$ , is less than unity. For the common emitter configuration the current ratio,  $\beta$ , may usually be of a value ranging from 10 to 50. The family of curves reminds us of a family of pentode characteristics. As with tubes, both a DC and an AC load line may be drawn. It is assumed here that the AC load line is so much steeper than the slope of the family of curves that the current transfer ratio, when loaded, is substantially the same as when the collector is AC short circuited. Thus, approximately, the current gain of the loaded transistor is  $\alpha$  for the common base circuit, and  $\beta$  for the common emitter circuit. The voltage gain

$$A_v = \frac{i_2 R_L}{i_1 r_i} = A_i \frac{R_L}{r_i} \quad (1)$$



is the current gain multiplied by the impedance ratio of load to input resistance.

### PERFORMANCE EQUATIONS

#### Common Base

$$A_i = \alpha$$

$$A_v = \frac{\alpha R_L}{r_{ib}}$$

$$G = A_i A_v = \frac{\alpha^2 R_L}{r_{ib}}$$

$$R_1 = r_{ib}$$

$$R_2 = r_{22b}$$

#### Common Emitter

$$A_i = \beta \quad (2)$$

$$A_v = \frac{\beta R_L}{r_{ie}} \quad (3)$$

$$G = A_i A_v = \frac{\beta^2 R_L}{r_{ie}} \quad (4)$$

$$R_1 = r_{ie} \quad (5)$$

$$R_2 = r_{22e} \quad (6)$$

No attention has been paid to signs, that is, relative directions and polarities.  $R_1$  is the input resistance.  $R_2$  is the output resistance.  $G$  is the power gain of the transistor. It is usually expressed in decibels. The average parameters of a 2N45 transistor are, in the common base circuit,

$$\alpha = .92$$

$$r_{ib} = 40 \text{ ohms}$$

$$r_{22b} = 1 \text{ megohm}$$



### CONVERSION OF PARAMETERS

It is not necessary to experimentally evaluate  $\beta$ ,  $r_{ie}$ , and  $r_{22e}$ . This may be done as follows with the help of figure 5.

$$\beta = \frac{i_c}{i_b} = \frac{\alpha i_e}{(1-\alpha) i_e}$$

$$\beta = \frac{\alpha}{1-\alpha} \quad (7)$$

To obtain  $r_{ie}$  in terms of  $r_{ib}$  and  $\alpha$ , use figures 6a and 6b, which show the AC measurement of input resistances.

If  $i_e$  is the same,  $i_b$  and  $v_1$  will also be the same in both circuits.

$$i_b = (1 - \alpha) i_e \quad (8)$$

$$r_{ib} = \frac{v_1}{i_e}$$

$$r_{ie} = \frac{v_1}{i_b}$$

$$r_{ie} = r_{ib} \frac{i_e}{i_b} = \frac{r_{ib}}{1-\alpha} \quad (9)$$

Though not so necessary, we can determine the output resistance of the common emitter configuration. Figure 7 shows the AC circuit for measuring the output impedance,  $r_{22b}$ .

Figure 8 shows the measurement of  $r_{22e}$ . The open base, (AC open ckt) may also be represented as carrying equal and opposite currents,  $(1 - \alpha) i_e$ . For the same voltage in both figures 7 and 8, the current  $i_b$  of figure 7 corresponds to the current  $(1 - \alpha) i_e$  entering the base in figure 8. This voltage-current ratio is the





collector barrier resistance  $r_{22b}$ . In figure 8, there is an additional current crossing the barrier,  $\alpha i_e$ . The total current crossing the barrier is  $i_e$ , so the output resistance is  $r_{22e} = \frac{v}{i_e}$ .

But  $r_{22b} = \frac{v}{(1-\alpha) i_e}$ , thus

$$r_{22e} = \frac{r_{22b}}{(1-\alpha)^{-1}} \quad (10)$$

Thus equations 7, 9 and 10 give the common emitter parameters in terms of those in the common base configuration.

In the common emitter circuit,

$$A_{ve} = \frac{\beta R_L}{r_{ie}} = \frac{\alpha}{1-\alpha} \cdot \frac{\frac{R_L}{\frac{r_{ib}}{1-\alpha}}}{\frac{r_{ib}}{1-\alpha}} = \frac{\alpha R_L}{r_{ib}} = A_{vb} \quad (11)$$

Thus for the same load, the transistor gives the same voltage gain in either the common base or common emitter circuit! However, the power gain is greater for the common emitter circuit because the voltage gain is multiplied by the current gain,  $\beta$  for the common emitter,  $\alpha$  for the common base circuit.

#### CASCADING OF R-C OR DIRECT COUPLED STAGES

For cascaded transistor stages, calculate gain for each stage using the appropriate load resistance. The input resistance of the succeeding stage becomes the load for the stage under consideration. Cascading common base stages is of no avail. The overall current gain is a fraction,  $\alpha^n$ , for  $n$  stages. The load resistance for all but the





last stage is the input resistance  $r_{ib}$ , so the voltage gain of each of these stages is

$$A_v = \alpha \frac{r_{ib}}{r_{ib}} = \alpha \quad (12)$$

Cascading common emitter stages gives current gain per stage of  $\beta$ . The voltage gain of each stage, except the output stage is

$$A_v = \frac{\beta r_{ie}}{r_{ie}} = \beta \quad (13)$$

The power gain per stage is  $G = \beta^2 \quad (14)$

If common base stages alternate with common emitter stages, it may be readily shown that for all but the last stage, the power gain of each is

$$G = \alpha \beta \quad (15)$$

For the common emitter stages the voltage gain is  $\alpha$ , and for the common base stages it is  $\beta$ .

In general, cascading will be done with common emitter stages, as that gives the greatest power gain from the input to the first transistor to the load at the output.

#### MISMATCH AT INPUT

Consider a source  $E$  of internal resistance  $R_s$ . Maximum power, called maximum available power, will be delivered to a matching load resistance.

$$M.A.P = \frac{E^2}{4R_s} \quad (16)$$



Into a mismatched input resistance,  $R_L$ , the power input is

$$P_L = \left( \frac{E}{R_S + R_L} \right)^2 R_L \quad (17)$$

$$\frac{P_L}{\text{M.A.P.}} = \frac{4 R_S R_L}{(R_S + R_L)^2} = \frac{4}{\frac{R_S + R_L}{R_L} \cdot \frac{R_S + R_L}{R_S}} \quad (18)$$

The mismatch loss in d.b. is

$$L = 10 \left[ \log \left( 1 + \frac{R_S}{R_L} \right) + \log \left( 1 + \frac{R_L}{R_S} \right) - \log 4 \right] \quad (19)$$

$$\text{where } \log 4 = .602$$

The amplifier power output with reference to the maximum available power from the signal source, is called the transducer gain. In decibels it is the sum of the transistor stage gains in db, minus the mismatch loss at the input.

Since there generally is a mismatch between the output impedance of one stage and the input impedance of the next, it might be thought that a mismatch loss must be taken into account between stages, as well as at the input. This is erroneous intuition, since the voltage gain (Eq. 3) was derived in terms of the load resistance.



## THE COMMON COLLECTOR or EMITTER FOLLOWER

Let us now consider the common collector circuit. The following simple treatment of the common collector circuit applies quite well for load resistance values between 300 ohms and 10,000 ohms. The AC component circuit is that of figure 10. It is seen that the transistor resistance which shunts the load is that between emitter and collector. This is the same as the  $r_{22e}$  of the common emitter circuit, hence for the low values of  $R_L$  as given above, we may neglect this shunting effect. According to this approximation then, the current gain is

$$A_i = \frac{i_2}{i_1} = \frac{i_e}{i_b} = \frac{i_e}{(1 - \alpha)i_e} = \frac{1}{1 - \alpha} = \beta + 1 \quad (20)$$

To consider voltage gain, it is apparent that  $v_2$  is only slightly less than  $v_1$  by the small AC voltage drop between emitter and base. Another way to express it is to say that, as the load sees it, there is only the small resistance,  $r_{ib}$ , in series with  $R_L$ . By figure 11 it is clear that the voltage gain is

$$A_v = \frac{R_L}{R_L + r_{ib}} \doteq 1 - \frac{r_{ib}}{R_L} \quad (21)$$

This shows that the term "emitter follower" is as descriptive for this circuit as "cathode follower" is for the corresponding tube circuit.

The power gain is

$$G = A_v A_i = (\beta + 1) \left(1 - \frac{r_{ib}}{R_L}\right) \doteq \beta \quad (22)$$





The input resistance of the transistor in this configuration is

$$R_1 = \frac{v_1}{i_1} = \frac{v_1}{(1 - \alpha)i_e} = \frac{R_L + r_{ib}}{1 - \alpha} \doteq \frac{R_L}{1 - \alpha} \doteq \beta R_L \quad (23)$$

This large input resistance is a unique feature of the common collector circuit; very much higher than that of the common base or common emitter circuits. The 100% series negative voltage feedback makes this high input resistance dependent on the load rather than on a transistor resistance.

As a result of feedback, the output impedance will be small and determined almost entirely by the equivalent resistance of the source. Figure 12 uses a generator in place of the load to calculate the output impedance. Neglecting the small AC voltage drop from emitter to base,  $v_e = v_b = i_b R_s$ . The output impedance is

$$R_2 = \frac{v_e}{i_e} = \frac{i_b R_s}{i_e} \doteq R_s (1 - \alpha) \doteq \frac{R_s}{\beta} \quad (24)$$

Another approximation involved above is that if  $R_s$  is not too large, then the emitter to collector shunt resistance  $r_{22e}$  may be neglected. Equations 23 and 24 show

$$R_1 R_2 \doteq R_s R_L \quad (25)$$

Equation 24 shows that a very low output impedance may be obtained if the source resistance is low. This is very important for using the transistor to regulate a power supply and is explained more fully later.





In resume, the emitter follower has close to unity voltage gain, current gain of  $\beta$ , nearly constant power gain of  $\beta$ , independent of loading, input resistance equal to  $\beta R_L$ , and output resistance equal to  $(1 - \alpha) R_s$ . Its usefulness in amplifiers will occur when the source has high impedance or when the load is of low resistance.

If an emitter follower is placed between a common base stage and a low resistance load, it will help greatly. By giving the common base (or a common emitter) stage a high resistance load,  $\beta$  times the actual load resistance, it will increase the gain of the preceding stage by nearly the  $\beta$  of the common collector stage.

One means of cascading an emitter follower with another stage is shown in figure 13.

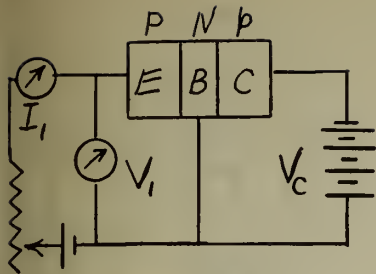
### PROBLEMS

1. Derive equation 15.
2. a. Show that the AC short circuit collector current of the common emitter circuit is to that of the common base circuit as  $\beta$  is to  $\alpha$ , if the AC input current is the same in both cases.  
b. Show that the open-circuit collector AC voltage has the same value in both configurations, for the same input.  
c. By the results above, derive equation 10.



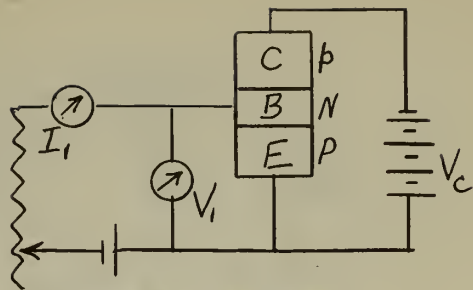
3. A two-stage transistor amplifier is to work between a 50 ohm signal source and a 2000 ohm load. Use the 2N45 transistor having  $\alpha = .92$   $r_{1b} = 40$  ohms and  $r_{22b} = 1$  megohm.
- a. Calculate the transducer gain when two common emitter stages are used.
  - b. Repeat for the input stage changed to common base.
  - c. Repeat for a common emitter input and a common base output stage.
  - d. Calculate the transducer gain when the first stage is a common base stage, and the second stage is an emitter follower. Let  $R$  of figure 13 be 6 k ohms.
  - e. For a 3 volt output, work through one of the amplifiers, calculating all AC voltages which could be checked by a VTVM.
4. Discuss the problem of attaining very low output impedance. Should the emitter follower have low or high  $\beta$ ? Should the preceding stage be a common base, common emitter, or common collector stage?





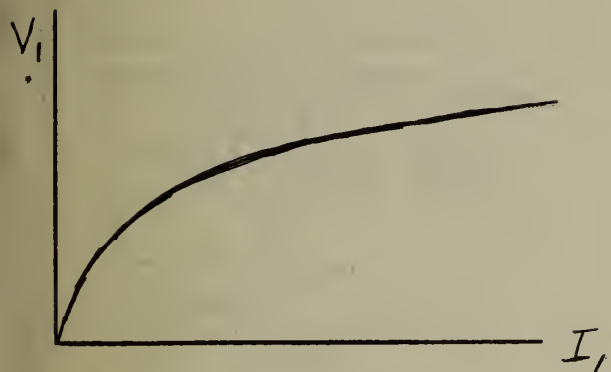
Common Base

Figure 1a



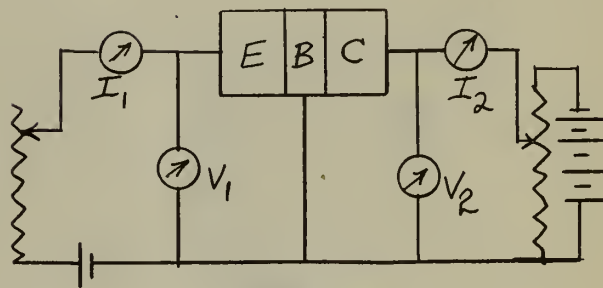
Common Emitter

Figure 1b



Input Characteristic

Figure 2



Common Base

Figure 3

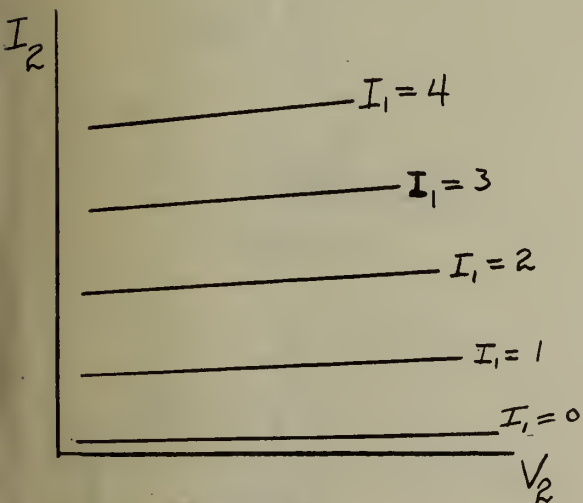


Figure 4

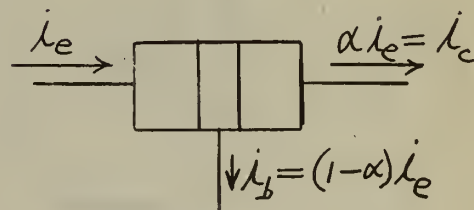


Figure 5

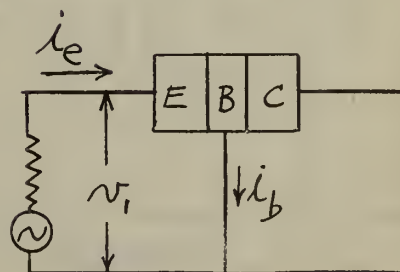


Figure 6a





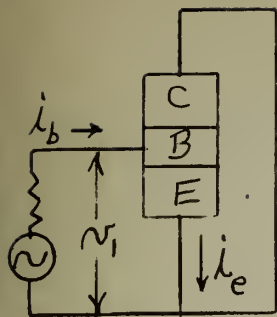


Figure 6b

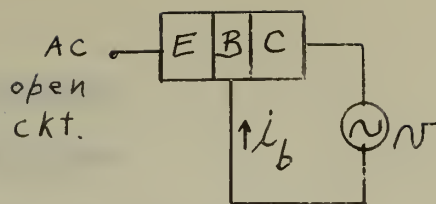


Figure 7

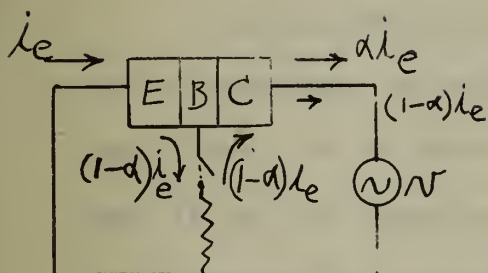


Figure 8

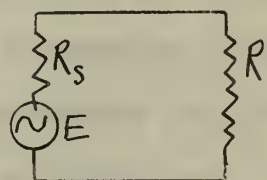


Figure 9

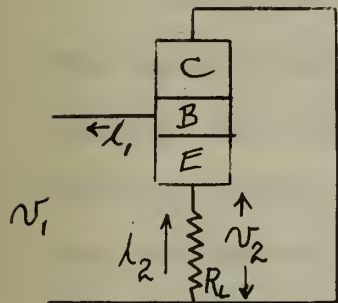


Figure 10

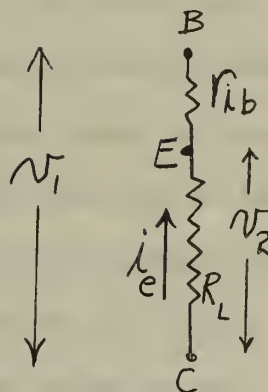


Figure 11

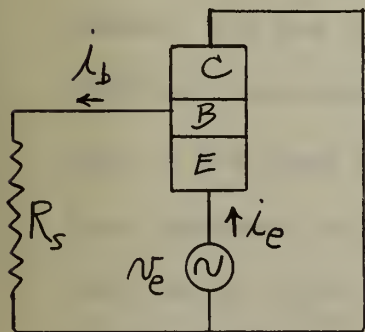


Figure 12

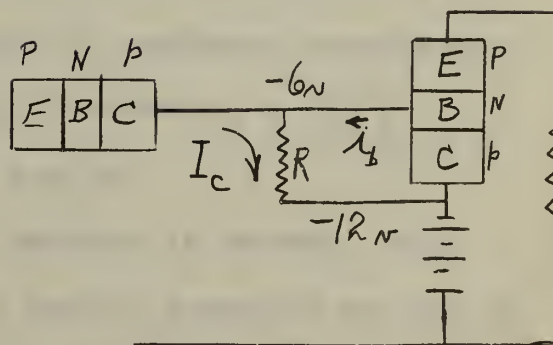


Figure 13





TRANSISTOR PARAMETERS  
and  
PERFORMANCE EQUATIONS

by

W. M. Bauer

October 1957

In the previous chapter a simplified treatment was presented of the parameters and performance equations of transistors as class A amplifiers. It was stated there that the approximations were very good if the load impedance is lower than 5,000 or 10,000 ohms. The justification for the simplified treatment is that it results in very simple equations which give all the accuracy that is warranted considering the range of values of parameters for a given transistor type.

While the majority of cases may be satisfactorily handled by ignoring the internal feedback, there are cases, such as transformer coupling, where the load impedance is so large that use of the exact equations becomes necessary. The first transistor parameters to be used were the open circuit resistance parameters\* which were satisfactory for point contact transistors. With the coming of the junction transistor, it became evident that a better system of parameters was needed. For one reason, the open circuit AC resistance parameters are difficult to measure. Consider trying to measure  $r_{11} = \left( \frac{v_1}{i_1} \right)_{i_2 = 0}$ . This is the input resistance measured when the load impedance relative to the output impedance is extremely large, so that the AC output current is zero. A junction transistor may have an output impedance of a megohm. How can you effectively provide an AC

\* Applied Electronics - Gray, p. 798



open circuit to a generator of internal impedance of 1 megohm?!

However, an easy measurement is the input resistance when the output is short circuited. It is easy to short circuit a high impedance, but difficult to open circuit a high impedance. Thus a load resistance of 100 ohms will effectively short circuit the output of a transistor. Calling  $r_i$  the input resistance when the output is shorted, this is one very satisfactory parameter.

The most widely used parameters are the hybrid or "h" parameters. One of the most distinctive features of a transistor is the dependence of the output current primarily upon the input current. In the previous chapter, the approximation was made that the output current depended on the input current only, for loads under 5 or 10 k. This dependence is both in magnitude and wave form. The independent variable for a transistor is the input current. Thus the most descriptive parameter of a transistor is its short circuit current transfer ratio.

Let us make a list of all the possible AC tests that could be made on a transistor amplifier stage. The box indicates the stage with the input shown on the plain side of the box, and the output indicated as the voltage into an open circuit or the current into a short circuit. The resulting parameters of each test are listed. It will be observed that, with the exception of some "a" and "b" parameters, the subscripts indicate where the data is taken. The order is that of numerator to denominator. The knowledge that it is a resistance or conductance parameter indicates whether the numerator is voltage or current. Unfortunately, the "h", "a" and "b" parameters give no hint as to whether









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they are a resistance, a conductance, or a ratio.

| <u>Tests</u>   | <u>Conductance</u>                                  | <u>Resistance</u>                                   |
|--|---|---|
| 1. $i_1 v_1$      | $y_{11} = \left( \frac{i_1}{v_1} \right)_{v_2 = 0}$ | $h_{11} = \left( \frac{v_1}{i_1} \right)_{v_2 = 0}$ |
| 2. $i_1 v_1$      | $g_{11} = \left( \frac{i_1}{v_1} \right)_{i_2 = 0}$ | $z_{11} = \left( \frac{v_1}{i_1} \right)_{i_2 = 0}$ |
| 3.  $i_2 v_2$     | $y_{22} = \left( \frac{i_1}{v_2} \right)_{v_1 = 0}$ | $g_{22} = \left( \frac{v_2}{i_2} \right)_{v_1 = 0}$ |
| 4.  $i_2 v_2$     | $h_{22} = \left( \frac{i_2}{v_2} \right)_{i_1 = 0}$ | $z_{22} = \left( \frac{v_2}{i_2} \right)_{i_1 = 0}$ |
| 5. $v_1$  $i_2$   | $y_{21} = \left( \frac{i_2}{v_1} \right)_{v_2 = 0}$ | $a_{12} = \left( \frac{v_1}{i_2} \right)_{v_2 = 0}$ |
| 6. $v_1$  $i_2$ | $b_{21} = \left( \frac{i_2}{v_1} \right)_{i_1 = 0}$ | $z_{12} = \left( \frac{v_1}{i_2} \right)_{i_1 = 0}$ |
| 7. $i_1$  $v_2$ | $a_{21} = \left( \frac{i_1}{v_2} \right)_{i_2 = 0}$ | $z_{21} = \left( \frac{v_2}{i_1} \right)_{i_2 = 0}$ |
| 8. $i_1$  $v_2$ | $y_{12} = \left( \frac{i_1}{v_2} \right)_{v_1 = 0}$ | $b_{12} = \left( \frac{v_2}{i_1} \right)_{v_1 = 0}$ |



# RATIOS

|     |   |   |   |
|-----|---|---|---|
| 9.  | $i_1$  $i_2$ | $h_{21} = \left( \frac{i_2}{i_1} \right)_{v_2 = 0}$ | $a_{22} = \left( \frac{i_1}{i_2} \right)_{v_2 = 0}$ |
| 10. | $i_1$  $i_2$ | $b_{22} = \left( \frac{i_2}{i_1} \right)_{v_1 = 0}$ | $g_{12} = \left( \frac{i_1}{i_2} \right)_{v_1 = 0}$ |
| 11. | $v_1$  $v_2$ | $g_{21} = \left( \frac{v_2}{v_1} \right)_{i_2 = 0}$ | $a_{11} = \left( \frac{v_1}{v_2} \right)_{i_2 = 0}$ |
| 12. | $v_1$  $v_2$ | $h_{12} = \left( \frac{v_1}{v_2} \right)_{i_1 = 0}$ | $b_{11} = \left( \frac{v_2}{v_1} \right)_{i_1 = 0}$ |

These parameters are all shown in neat matrix form in Shea (T.C.E., p. 22) and all interrelations among the six sets of parameters are given in Shea (T.C.E., p. 440-444). It is a rather overwhelming array and one feels a nostalgia for the vacuum tube which has only three parameters (coefficients) as against twenty-four for the transistor. Actually there are only twelve distinct parameters. But this comes up to thirty-six when we consider that there are the three transistor configurations of common emitter, common base, and common collector!

The "h" system is really a hybrid, as we see it picks one parameter out of each of the "y", "z", "a", and "b" parameters and is distinctive from them only in being the reciprocal. The selection of what set of four parameters to use is based largely on the ease of AC measurement, as was mentioned earlier. Measurements made with the





output short circuit<sup>ed</sup> are good ones because it is easy to AC short the large output impedance. Thus tests 1, ~~8~~<sup>5</sup> and 9 would be good choices. At the input an AC open circuit is easy to obtain so that tests 4, 6 and 12 are indicated.

|                           |          |          |          |          |          |
|---------------------------|----------|----------|----------|----------|----------|
| Tests                     |          |          |          |          |          |
| 1                         | $y_{11}$ | $h_{11}$ |          |          |          |
| <del>8</del> <sup>5</sup> | $y_{21}$ |          | $a_{12}$ |          |          |
| 9                         |          | $h_{21}$ | $a_{22}$ |          |          |
| 4                         |          | $h_{22}$ |          | $z_{22}$ |          |
| 6                         |          |          |          | $z_{12}$ | $b_{21}$ |
| 12                        |          | $h_{12}$ |          |          | $b_{11}$ |

By the above tabulation the "h" system is the proper choice for ease of measurement. The  $h_{12}$  parameter is the most difficult to measure since it is very small and the open circuit voltage may get down in the noise voltage and be masked by pick-up. The parameter  $z_{11}$  would be a nice companion to  $h_{11}$  as the actual input resistance of an amplifier would always be somewhere between these two parameters. Likewise  $y_{22}$  together with  $h_{22}$  would show between them wherein the output conductance of an amplifier could lie. Parameter  $z_{11}$  is impossible to measure, but  $y_{22}$  may not be too difficult to measure.

To begin the analysis of the hybrid system, figure 1 shows a transistor with actual currents and voltages. Figure 2 is a formalized version of figure 1 to apply to either a PNP or NPN transistor in any configuration. Directions of currents and polarities of voltages are arbitrarily assigned. The "h" system says the AC input voltage and output current are functions of the input current and the output voltage.



$$V_1 = F_1(I_1, V_2) \quad (1)$$

$$I_2 = F_2(I_1, V_2) \quad (2)$$

The incremental equations are,

$$\Delta I_2 = \left. \frac{\partial I_2}{\partial I_1} \right|_{V_2 \text{ const.}} \cdot \Delta I_1 + \left. \frac{\partial I_2}{\partial V_2} \right|_{I_1 \text{ const.}} \cdot \Delta V_2 \quad (3)$$

$$\Delta V_1 = \left. \frac{\partial V_1}{\partial I_1} \right|_{V_2 \text{ const.}} \cdot \Delta I_1 + \left. \frac{\partial V_1}{\partial V_2} \right|_{I_1 \text{ const.}} \cdot \Delta V_2 \quad (4)$$

The AC component of these equations is,

$$i_2 = \left. \frac{i_2}{i_1} \right|_{V_2 = 0} \cdot i_1 + \left. \frac{i_2}{V_2} \right|_{i_1 = 0} \cdot v_2 \quad (5)$$

$$v_1 = \left. \frac{v_1}{i_1} \right|_{V_2 = 0} \cdot i_1 + \left. \frac{v_1}{V_2} \right|_{i_1 = 0} \cdot v_2 \quad (6)$$

Figure 2 becomes figure 3 for AC.

The meaning of  $v_2 = 0$  is that there is an AC short circuit across the output terminals. And  $i_1 = 0$  means that there is infinite impedance for AC in the input circuit. The h-parameters are the coefficients in the equations above.

Equations 5 and 6 may be written,

$$i_2 = h_{21} \cdot i_1 + h_{22} \cdot v_2 \quad (7)$$

$$v_1 = h_{11} \cdot i_1 + h_{12} \cdot v_2 \quad (8)$$





$h_{11}$  is the input resistance when the output is AC short-circuited, usually termed the short-circuited input resistance. This parameter was called  $r_i$  in the previous chapter. Another symbol is  $h_i$ . For most common base and common emitter circuits the input resistance is very close to the value of this parameter.

$h_{12}$  is the ratio of the voltage at the open-circuited input to the voltage applied to the output. It is called the reverse voltage feedback ratio. Its effect is small when the load resistance is not too large. Another symbol is  $h_r$ .

$h_{21}$  is the ratio of the current in the short-circuited output to the current introduced into the input. It is the forward AC current amplification ratio. This is no doubt the most important and most defining of the transistor parameters. Its absolute value was called  $\alpha$  for the common base, and  $\beta$  for the common emitter configurations in the previous chapter. Another general symbol is  $h_f$ . The current gain of most amplifiers is only slightly less than the value of this parameter.

$h_{22}$  is the output conductance when the input circuit is AC open-circuited. It is the open-circuited output conductance, and is the reciprocal of  $r_{22}$  in the previous chapter. Another symbol is  $h_o$ .

An objection to the h-parameters is that the letter h means nothing to identify the dimensions of the unit, so other letters will be used to indicate resistance.

$$r_i = h_{11} \quad \text{and} \quad r_{22} = \frac{1}{h_{22}}$$





Equations 7 and 8 become,

$$i_2 = h_{21} \cdot i_1 + \frac{v_2}{r_{22}} \quad (9)$$

$$v_1 = r_1 \cdot i_1 + h_{12} \cdot v_2 \quad (10)$$

Now these equations may be interpreted as the AC equivalent circuit of figure 4.

The approximations of the previous chapter may be seen to be the omissions of the last terms of equations 9 and 10, which is the omission of the feedback generator in the input circuit, and the internal shunt in the output circuit. This equivalent circuit will now be drawn inside the transistor of figure 1, and the AC currents and voltages shown in figure 5.

The forward biased PN junction offers only a small body resistance  $r_1$  and the small counter voltage  $h_{12}v_2$ . If the output is AC short-circuited, there is only  $r_1$  between emitter and base. If  $R_L$  causes the collector to become less negative when the emitter current increases, it is to be expected that the emitter current is going to be slightly less than if the collector potential had not decreased. Thus the feedback voltage  $h_{12}v_2$  opposes  $i_e$  and causes  $v_{eb}$  to be larger. This is negative voltage feedback.

The collector junction is equivalent to a very large resistance  $r_{22}$ , shunted by a constant current generator which is the real heart of transistor action. This current generator is  $h_{21}i_1$  which in figure 5 for the common base circuit is replaced by  $\alpha i_e$ . The short-circuited current ratio,  $\frac{i_c}{i_e}$ , in the common base circuit is designated



by  $\alpha$  and since the AC collector current flows out when the AC emitter current flows in,  $\alpha = -h_{21b}$ . This is to say that  $\alpha$  is a positive number like .95 and  $h_{21b}$  is  $-.95$ . Equations 9 and 10 become

$$i_c = \alpha \cdot i_e - \frac{v_{cb}}{r_{22b}} \quad (11)$$

$$v_{eb} = r_{1b} i_e + h_{12b} v_{cb} \quad (12)$$

Inasmuch as the h parameters are quite thoroughly entrenched in the literature, let us return to equations 7 and 8 to obtain the performance equations in terms of the h parameters. If  $v_2$ , of figure 3, is due to a load,  $R_L$ , then

$$v_2 = -i_2 \cdot R_L \quad (13)$$

Equation 7 becomes

$$i_2 = h_{21} \cdot i_1 - h_{22} \cdot R_L \cdot i_2 \quad (14)$$

which may be solved for

$$i_2 = \frac{h_{21} i_1}{1 + h_{22} R_L} \quad (15)$$

The current gain of the stage is

$$A_i = \frac{i_2}{i_1} = \frac{h_{21}}{1 + h_{22} R_L} \quad (16)$$

If equation 13 is put into equation 8, the input resistance is

$$\frac{v_1}{i_1} = R_1 = h_{11} - h_{12} \left( \frac{i_2}{i_1} \right) R_L \quad (17)$$

1. The first part of the problem is to find the value of  $\frac{dy}{dx}$  at the point  $(1, 2)$  on the curve  $y = x^2 + 3x - 5$ . This can be done by differentiating the equation with respect to  $x$ .

$$\frac{dy}{dx} = 2x + 3$$

$$\frac{dy}{dx} = 2(1) + 3 = 5$$

2. The second part of the problem is to find the equation of the tangent line to the curve  $y = x^2 + 3x - 5$  at the point  $(1, 2)$ . We know that the slope of the tangent line is  $\frac{dy}{dx}$  at that point, which is 5. Using the point-slope form of a line, we can find the equation of the tangent line.

$$y - 2 = 5(x - 1)$$

3. The third part of the problem is to find the area under the curve  $y = x^2 + 3x - 5$  from  $x = 0$  to  $x = 2$ . This can be done by integrating the function with respect to  $x$ .

$$\int_0^2 (x^2 + 3x - 5) dx = \left[ \frac{x^3}{3} + \frac{3x^2}{2} - 5x \right]_0^2$$

4. The fourth part of the problem is to find the volume of the solid generated by revolving the curve  $y = x^2 + 3x - 5$  around the  $y$ -axis from  $x = 0$  to  $x = 2$ . This can be done by using the disk method.

$$V = \pi \int_0^2 (x^2 + 3x - 5)^2 dx$$

5. The fifth part of the problem is to find the length of the arc of the curve  $y = x^2 + 3x - 5$  from  $x = 0$  to  $x = 2$ . This can be done by using the arc length formula.

$$L = \int_0^2 \sqrt{1 + (2x + 3)^2} dx$$

6. The sixth part of the problem is to find the average value of the function  $f(x) = x^2 + 3x - 5$  on the interval  $[0, 2]$ . This can be done by using the average value formula.

$$f_{\text{avg}} = \frac{1}{2} \int_0^2 (x^2 + 3x - 5) dx$$

and by equation 16

$$R_1 = h_{11} - h_{12}A_i R_L \quad (18)$$

The power input is  $i_1^2 R_1$ , and the power output is  $i_2^2 R_L$ , so the power gain of the transistor is

$$G = \frac{R_L}{R_1} \cdot (A_i)^2 = |A_i A_v| \quad (19)$$

The voltage gain is

$$A_v = \frac{v_2}{v_1} = \frac{-i_2 R_L}{i_1 R_1} = -\frac{A_i R_L}{R_1} \quad (20)$$

The output admittance by equation 7 is

$$Y_2 = \frac{i_2}{v_2} = h_{22} + \frac{h_{21}i_1}{v_2} \quad (21)$$

where  $v_2$  is an applied voltage in figure 3, and  $i_1$  is the current flowing into the input terminal of the transistor as a result of the internal feedback voltage which is introduced into the input circuit due to the output voltage. This feedback voltage is the last term of equation 8. The current it causes in the input circuit is to be found from equation 8 with  $v_1 = -i_1 R_s$ , where  $R_s$  is the source resistance.

$$v_1 = -i_1 R_s = h_{11}i_1 + h_{12}v_2 \quad (22)$$

whence

$$\frac{i_1}{v_2} = \frac{-h_{12}}{R_s + h_{11}}$$

which substituted in equation 21 gives

$$Y_2 = h_{22} - \frac{h_{12} \cdot h_{21}}{R_s + h_{11}} \quad (23)$$



$$f(x) = x^2 + 1$$

The first term is  $f'(x)$ , and the second term is  $f(x)$ . The third term is  $f''(x)$ , and the fourth term is  $f(x)$ .

$$\left( \frac{d}{dx} \right)^2 f(x) = \frac{d^2}{dx^2} f(x) = 2x$$

The second term is

$$\frac{d^2}{dx^2} f(x) = \frac{d^2}{dx^2} (x^2 + 1) = 2x$$

The third term is  $f''(x)$ , and the fourth term is  $f(x)$ .

$$\frac{d^2}{dx^2} f(x) = 2x + \frac{d^2}{dx^2} f(x) = 2x$$

Since  $f(x) = x^2 + 1$ , we have  $f'(x) = 2x$  and  $f''(x) = 2$ . The first term is  $f'(x)$ , and the second term is  $f(x)$ . The third term is  $f''(x)$ , and the fourth term is  $f(x)$ . The first term is  $f'(x)$ , and the second term is  $f(x)$ . The third term is  $f''(x)$ , and the fourth term is  $f(x)$ . The first term is  $f'(x)$ , and the second term is  $f(x)$ . The third term is  $f''(x)$ , and the fourth term is  $f(x)$ .

$$f(x) = (x^2 + 1) + 2x + 2$$

where

$$\frac{d^2}{dx^2} f(x) = \frac{d^2}{dx^2} (x^2 + 1) = 2x$$

and the third term is  $f''(x)$ , and the fourth term is  $f(x)$ .

$$\frac{d^2}{dx^2} f(x) = 2x + \frac{d^2}{dx^2} f(x) = 2x$$



Equation 16 may be written  $A_i = h_{21} \cdot \overline{R_L}^1$  (24)

where  $\overline{R_L}^1$  is the parallel resistance of  $R_L$  and  $r_{22}$  in figure 4.

Also equation 20,  $A_v = \frac{-h_{21} \overline{R_L}^1}{R_1}$  (25)

Below are listed the performance equations in the order in which they are used in calculation. The approximate equations of the previous chapter are shown for comparison.

h-equations approximate equations

$$A_i = h_{21} \frac{R_L^1}{R_L} \quad A_i = h_{21} \quad (26)$$

$$R_1 = h_{11} - h_{12} \cdot h_{21} \cdot R_L^1 \quad R_1 = h_{11} \quad (27)$$

$$A_v = \frac{-h_{21} \cdot R_L^1}{R_1} \quad A_v = \frac{-h_{21} \cdot R_L}{h_{11}} \quad (28)$$

$$\frac{1}{R_2} = h_{22} \cdot \left( \frac{h_{12} \cdot h_{21}}{R_s + h_{11}} \right) \quad \frac{1}{R_2} = h_{22} \quad (29)$$

Actually the calculation by the exact h-equations is simple, and the amount of the correction to the approximate equation is apparent.

As previously mentioned,  $h_{12}$ , may be difficult to measure due to noise and pickup. If we make test #3 p. 3, we get parameter  $g_{22}$ , the short circuited output resistance which I shall call  $r_o$ . By equation 29,

$$\frac{1}{r_o} = h_{22} - \frac{h_{12} \cdot h_{21}}{h_{11}} \quad (30)$$

Let  $f(x) = \frac{1}{x}$  and  $g(x) = \frac{1}{x^2}$ . Then  $f(x)g(x) = \frac{1}{x^3}$ .  
 The derivative of  $f(x)g(x)$  is  $-\frac{3}{x^4}$ .  
 The derivative of  $f(x)$  is  $-\frac{1}{x^2}$ .  
 The derivative of  $g(x)$  is  $-\frac{2}{x^3}$ .  
 The product rule states that  $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ .  
 Substituting the derivatives, we get  $-\frac{3}{x^4} = -\frac{1}{x^2} \cdot \frac{1}{x^2} + \frac{1}{x} \cdot -\frac{2}{x^3}$ .  
 Simplifying the right side, we get  $-\frac{1}{x^4} - \frac{2}{x^4} = -\frac{3}{x^4}$ .  
 This confirms the product rule for this example.

Example 2:

$$f(x) = x^2, g(x) = x^3 \Rightarrow f(x)g(x) = x^5$$

$$f'(x) = 2x, g'(x) = 3x^2 \Rightarrow (f(x)g(x))' = 5x^4$$

$$\frac{d}{dx}(x^2 \cdot x^3) = \frac{d}{dx}(x^5) = 5x^4$$

$$= 2x \cdot x^3 + x^2 \cdot 3x^2 = 2x^4 + 3x^4 = 5x^4$$

Example 3: Let  $f(x) = \sin(x)$  and  $g(x) = \cos(x)$ .  
 Then  $f(x)g(x) = \sin(x)\cos(x) = \frac{1}{2}\sin(2x)$ .  
 The derivative of  $f(x)g(x)$  is  $\cos(2x)$ .  
 The derivative of  $f(x)$  is  $\cos(x)$ .  
 The derivative of  $g(x)$  is  $-\sin(x)$ .  
 The product rule states that  $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ .  
 Substituting the derivatives, we get  $\cos(2x) = \cos(x)\cos(x) + \sin(x)(-\sin(x))$ .  
 Simplifying the right side, we get  $\cos^2(x) - \sin^2(x) = \cos(2x)$ .  
 This confirms the product rule for this example.

This may be solved, giving,

$$h_{12} = \frac{h_{11}}{h_{21}} \left( h_{22} - \frac{1}{r_o} \right) \quad (31)$$

Thus to determine  $h_{12}$  we may measure  $r_o$  and avoid direct measurement of  $h_{12}$ . Calculations may be made by equations 26-29, or equations 27 and 29 may be solved by substitution of equation 31.

$$R_1 = h_{11} \cdot \frac{1 + \frac{R_L}{r_o}}{1 + \frac{R_L}{r_{22}}} \quad (32)$$

$$R_2 = r_{22} \cdot \frac{1 + \frac{h_{11}}{R_s}}{1 + \frac{h_{11} \cdot r_{22}}{R_s r_o}} \quad (33)$$

$$A_v = \frac{-h_{21} \cdot R_L}{h_{11} \left( 1 + \frac{R_L}{r_o} \right)} \quad (34)$$

$$A_i = \frac{h_{21}}{1 + \frac{R_L}{r_{22}}} \quad (35)$$

$$G = |A_v \cdot A_i| = \frac{h_{21}^2 \cdot r_o \cdot r_{22}}{h_{11}} \cdot \frac{R_L}{(r_{22} + R_L)(r_o + R_L)} \quad (36)$$



Differentiation of equation 36 yields the condition for maximum power output,

$$R_L = \sqrt{r_o \cdot r_{22}} \quad (37)$$

Thus a load resistance which is the geometric mean of the short and open circuited output resistances gives maximum power output for a given value of power input to the transistor. The maximum power gain by 36 and 37 is,

$$G_{\max} = \frac{h_{21}^2 \sqrt{r_o \cdot r_{22}}}{h_{11}} \cdot \frac{1}{\left(1 + \sqrt{\frac{r_o}{r_{22}}}\right) \left(1 + \sqrt{\frac{r_{22}}{r_o}}\right)} \quad (38)$$

By equation 32, the input resistance for the condition of maximum power gain is

$$R_1 = h_{11} \sqrt{\frac{r_{22}}{r_o}} \quad (39)$$

By equation 32 the maximum input resistance is when  $R_L$  is infinite. This is the parameter  $Z_{11}$  of test #2 which is impossible to measure directly. It will be called  $r_{11}$ .

$$r_{11} = h_{11} \frac{r_{22}}{r_o} \quad (40)$$

In equation 33 we see this term, so

$$R_2 = r_{22} \cdot \frac{1 + \frac{h_{11}}{R_s}}{1 + \frac{r_{11}}{R_s}} \quad (41)$$

The open circuit voltage gain by equation 28 or 34 is,

$$A_{Voc} = -h_{21} \frac{r_{22}}{r_{11}} = -h_{21} \frac{r_o}{h_{11}} \quad (42)$$





where  $h_{21}$  is the short circuit current gain.

The maximum gain equation 38 may be written,

$$G_{\max} = -h_{21} \left( \frac{-h_{21} \cdot r_o}{h_{11}} \right) \cdot \frac{1}{\left( 1 + \sqrt{\frac{r_o}{r_{22}}} \right)^2} \quad (43)$$

This may be interpreted as the product of the short circuit current gain and the open circuit voltage gain multiplied by the reduction factor.

By equations 39 and 40 the value of  $R_1$  for maximum power gain is,

$$R_1 = \sqrt{h_{11} \cdot r_{11}} \quad (44)$$

It is similar to equation 37, in that it is the geometric mean of the open and short circuit input resistances.

Now if the source resistance happens to match this value of  $R_1$ , then the maximum available power is obtained from the source, and the optimum power output of the combination of source and transistor is realized. As stated in the previous chapter, the amplifier power output with reference to the maximum available power from the source is called the transducer gain.

$$G_t = \frac{P_o}{\text{M.A.P.}} = \frac{P_o}{P_1} \cdot \frac{P_1}{\text{M.A.P.}} = G \frac{P_1}{\text{M.A.P.}} \quad (45)$$

The second term was discussed in the previous chapter as a mismatch loss. So the maximum transducer gain is

$$G_{t_{\max}} = G \quad (46)$$

$$G_{t_{\text{opt}}} = G_{\max} \quad (47)$$



Optimum transducer gain occurs when  $R_L = \sqrt{r_o \cdot r_{22}}$  and when  $R_S = \sqrt{h_{11} \cdot r_{11}}$ . It is a condition seldom realized.

#### SUMMARY OF PARAMETERS

| Test # |   |   | Other symbols        |
|--------|---|---|----------------------|
| 9      | current gain<br>(output AC shorted)                   | $h_{21}$  | $\alpha, \beta, h_f$ |
| 4      | output resistance<br>(input AC open)                  | $r_{22} = \frac{1}{h_{22}}$   | $\frac{1}{h_o}$      |
| 3      | output resistance<br>(input AC shorted)               | $r_o$   |                      |
| 1      | input resistance<br>(output AC shorted)               | $r_i = h_{11}$  | $h_i$                |
|        | input resistance<br>(output AC open)                  | $r_{11} = \frac{h_{11} \cdot r_{22}}{r_o}$                                    |                      |
| 12     | internal feedback<br>voltage ratio<br>(input AC open) | $h_{12} = \frac{r_i}{h_{21}} \left( \frac{1}{r_{22}} - \frac{1}{r_o} \right)$ | $h_r$                |

The first four are the parameters which are most practical for measurement, and the last two are calculated.



# SUMMARY OF PERFORMANCE EQUATIONS

Approximate

$$48) \quad A_i = \frac{h_{21}}{1 + \frac{R_L}{r_{22}}} = \frac{h_{21}}{1 + h_{22}R_L} \quad A_i = h_{21}$$

$$49) \quad R_i = r_i \cdot \frac{1 + \frac{R_L}{r_o}}{1 + \frac{R_L}{r_{22}}} = h_{11} - \left( \frac{h_{12} \cdot h_{21} \cdot R_L}{1 + h_{22} R_L} \right) \quad R_i = h_{11}$$

$$50) \quad A_v = \frac{-A_i \cdot R_L}{R_i} = \frac{-h_{21} \cdot R_L}{r_i \left( 1 + \frac{R_L}{r_o} \right)} = \frac{-h_{21}}{R_i} \cdot \frac{R_L}{1 + h_{22} R_L} \quad A_v = \frac{-h_{21} R_L}{h_{11}}$$

$$51) \quad R_2 = r_{22} \cdot \frac{1 + \frac{r_i}{R_s}}{1 + \frac{r_{11}}{R_s}} ; \quad \frac{1}{R_2} = h_{22} - \frac{h_{12} \cdot h_{21}}{R_s + h_{11}} \quad R_2 = \frac{1}{h_{22}}$$

$$52) \quad G = -A_v \cdot A_i$$

Input resistance will have values, determined by  $R_L$ , which will be between the parameter values of  $r_i$  and  $r_{11}$ . Output resistance will have values, determined by  $R_s$ , which will be between the parameter values of  $r_{22}$  and  $r_o$ .





## COMMON BASE CIRCUIT

The common base circuit has already been shown in figures 1 and 5.

It was shown that  $\alpha = -h_{21b}$ , and equations 11 and 12 are for the common base configuration, for the directions of currents and polarities shown in figures 1 and 5.

The experimental static characteristics are shown in figures 7 and 8. Figure 7 shows the collector characteristics of the grounded or common base circuit. Figure 8 gives the emitter characteristics. The feedback effect is indicated by the separation of the curves. Actually this separation is shown considerably exaggerated here. If the collector is made less negative, it will require more emitter voltage to maintain the emitter current constant.

Equation 11 is 
$$i_c = \alpha \cdot i_e - \frac{v_{cb}}{r_{22b}}$$

The first term is shown in figure 7 as ac. If  $V_{cb}$  were constant ( $v_{cb} = 0$ ),  $I_c$  would increase from a to c by  $(\alpha \cdot i_e)$ . But the collector potential drops as  $I_c$  increases and the terminal point b on the load line is reached. The second term of equation 11 is shown as d-b in figure 7, since c-d represents  $v_{cb}$ , and the slope of the characteristic curves is the reciprocal of  $r_{22b}$ .

Equation 12 is, 
$$v_{eb} = r_{1b} \cdot i_e + h_{12b} \cdot v_{cb}$$

It is similarly interpreted in figure 8. The slope of the emitter characteristic is  $r_{1b}$ , so the first term of 12 is shown as c-d. The second term is represented by c-b.  $h_{12b}$  is a positive quantity, since



the emitter goes more positive as the collector swings positively. This internal feedback is negative series voltage feedback. It is negative since the input voltage must be larger than without this feedback. The dynamic path from a to b in figure 8 is steeper than the path a to c, which would be the path without feedback. Thus feedback increases the input impedance.

It might seem that the static characteristic curves would be used, as they are for tubes, to evaluate parameters. It may be done as shown above and in problem 3, but the static curves furnished by most manufacturers are too small to be of much use. Also the accuracy of obtaining the static data is insufficient, and the separation of the input characteristics is so slight that this method is practically never used. The parameters are determined by AC measurements.

### COMMON EMITTER

A transistor is shown in the common emitter circuit in figure 9. The DC polarities and currents are shown by capital letters. The actual incremental polarities and current directions are shown by small letters. Figure 10 shows the common-emitter, collector static characteristics, and figure 11 shows the base input characteristics. By comparing the AC voltages and currents of figure 9 with the formalized diagram of figure 3, it is observed that only  $v_{ce}$  is opposite to the assigned polarity  $v_2$ .

Hence in writing equations 9 and 10,  $v_2$  must be replaced by  $-v_{ce}$ .





$$i_c = h_{21e} \cdot i_b + \frac{(-v_{ce})}{r_{22e}}$$

or

$$i_c = \beta i_b - \frac{v_{ce}}{r_{22e}} \quad (9)$$

Figure 10 shows these two components of equation 9. Equation 10 for the common emitter circuit is,

$$v_{be} = r_{1e} \cdot i_b - h_{12e} \cdot v_{ce} \quad (10)$$

In going from a to b we see that  $v_{be}$  is the difference of the terms of equation 10. From a to b,  $v_{ce}$  is positive,  $i_b$  is positive and so both terms of equation 10 are positive which requires  $h_{12e}$  to be positive. Figure 11 evaluates  $h_{12e}$  by going vertically from one curve to the other;  $v_{ce}$  is positive in moving down. This decreases the negative  $V_{be}$  which is a positive  $v_{be}$ . Thus  $h_{12e}$  is positive. Is the internal feedback negative or positive? In going from a to b in figure 11, the change of  $V_{be}$  is less than if  $V_{ce}$  remained constant. If less AC input voltage is needed, then the feedback is positive. The decrease of slope of the dynamic path shows lowered input resistance resulting from positive series voltage feedback.

### COMMON COLLECTOR

A transistor in a common collector circuit is shown in figure 12. Figure 13 is the AC equivalent. It shows clearly that the collector is common to the two current loops. The AC emitter current is opposite to the assigned positive direction of  $i_2$  in figure 3, hence  $h_{21c}$  is a negative quantity. Figure 14 shows how the emitter follows the potential of the base. This circuit has 100% series negative voltage





feedback in the external circuit, which so completely swamps the small internal feedback that  $h_{12c}$  is taken as unity, positive.

CONVERSION OF PARAMETERS  
from  
ONE CONFIGURATION TO ANOTHER

It is unnecessary that four parameters be measured for each of the three circuits. Having obtained one set, all others may be calculated if the interrelationships are developed.

First consider the short-circuited current ratios. Figure 15 is drawn with all terminals shorted. To consider it as a common base circuit, imagine a constant current generator in the emitter lead.

$\alpha = \frac{i_c}{i_e}$ . As a common emitter circuit, imagine the constant current generator in the base lead. Then,

$$\beta = \frac{i_c}{i_b} = \frac{\alpha}{1 - \alpha} \quad (53)$$

As a common collector circuit,  $i_b$  is the constant current input again, but  $i_e$  is the output. Thus,

$$h_{21c} = \frac{i_e}{i_b} = \frac{1}{1 - \alpha} = \frac{\alpha + 1 - \alpha}{1 - \alpha} = \beta + 1 \quad (54)$$

Next consider the input open-circuited resistance. From figures 16 and 17, it is obvious that both have the same input resistance.

So,  $r_{11b} = r_{11e} \quad (55)$

Likewise figures 18 and 19 show

$$r_{11c} = r_{22b} \quad (56)$$



Next consider the input short-circuited resistances.

Figure 20 shows

$$r_{ic} = r_{ie} \quad (57)$$

By reference to figure 15, the AC voltage between emitter and base has one particular value whether  $i_e$  or  $i_b$  is the input. The input resistances will then be in the inverse ratio of these currents. Thus

$$r_{ie} = \frac{r_{ib}}{1 - \alpha} \quad (58)$$

Consider the output short-circuited resistances. Figures 21 and 22 show

$$r_{oe} = r_{ob} \quad (59)$$

Considering figures 23 and 24, it is clear that

$$r_{oc} = r_{ib} \quad (60)$$

In figures 25 and 26, it is seen that

$$r_{22c} = r_{22e} \quad (61)$$

In the previous chapter it was shown that

$$r_{22e} = \frac{r_{22b}}{1 - \alpha} \quad (62)$$



# PARAMETER CONVERSION RELATIONS

|                                   | Common base   | Common emitter   | Common collector                       |
|-----------------------------------|---|--|--|
| $h_{21}$                          | -   | +  | -                                      |
| $ h_{21} $                        | $\alpha = \frac{\beta}{\beta + 1}$                        | $\beta = \frac{\alpha}{1 - \alpha}$                      | $\gamma = \beta + 1$                   |
| $1/h_{22}$                        | $r_{22b} = (\beta + 1)r_{22e}$                            | $r_{22e} = \frac{r_{22b}}{\beta + 1}$                    | $r_{22c} = r_{22e}$                    |
| $r_o$                             | $r_{ob} = r_{oe}$   | $r_{oe} = r_{ob}$  | $r_{oc} = r_{ib}$                      |
| $h_{11}$                          | $r_{ib} = \frac{r_{ie}}{\beta + 1}$                       | $r_{ie} = r_{ib}(\beta + 1)$                             | $r_{ic} = r_{ie}$                      |
| $r_{11} = \frac{r_i r_{22}}{r_o}$ | $r_{11b} = r_{11e}$                                       | $r_{11e} = r_{11b}$                                      | $r_{11c} = r_{22b}$                    |
| $h_{12}$                          | $h_{12b} = \frac{r_{11b} - r_{ib}}{\alpha \cdot r_{22b}}$ | $h_{12e} = \frac{r_{ie} - r_{11e}}{\beta \cdot r_{22e}}$ | $h_{12c} = 1 - \frac{r_{oc}}{r_{22c}}$ |

The first four parameters are the ones to be measured since it is impossible to measure  $r_{11}$  and difficult to measure  $h_{12}$ . Consideration of the conversion relations shows that the best single configuration for measurement is the common emitter. Suppose  $\alpha$  were measured to 1% accuracy, then  $1 - \alpha$  and  $\beta$  would be obtained accurate only to 20% since  $\alpha$  is very close to unity. Thus the current transfer ratio,  $h_{21}$ , should be measured in the common emitter circuit. The value of  $r_{22e}$  is not as large as  $r_{22b}$  and should be more easily measured. The





ratio of  $r_i$  to the biasing input resistance will be about the same for either configuration, so the common emitter circuit is good for measurement of  $r_{22}$ . For measurement of  $r_o$  the common emitter is decidedly superior, for it is easier to short its higher value of input resistance. Measurement of  $r_i$  should be easy with either circuit.

If a set of h-parameters is supplied by the manufacturer, one may apply the conversion equations 63, 64, 66 and 68 to obtain sets of h-parameters in any desired configuration. Performance may be calculated by the equations 48-52.

If it is ever desired to convert the h-parameters to any of the other systems, z, y, a, or b, this may be done. For illustration, take the y-parameters where

$$I_1 = f_1 (V_1, V_2) \quad (69)$$

$$I_2 = f_2 (V_1, V_2) \quad (70)$$

Treat these as we did for the h-parameters through equations 1 - 8.

By comparison of the differential equations, it can be seen directly that  $y_{11} = h_{11}$ . The remaining relations may be obtained by the mathematical manipulation of partial differentials.



## EQUIVALENT CIRCUITS

In figure 4 is the low frequency AC h-equivalent circuit.

Figure 5 shows how the elements of the circuit relate to the transistor. Another equivalent circuit widely used is the active T equivalent of figure 27 as it is usually shown for the common base circuit. However, for generality it is shown in figure 28. This is the AC equivalent circuit we are assigning to the box of figure 3. The T circuit for the common base configuration of figure 27 has some very commendable features. It has a current generator delivering current to the output which is a function of the input current. The resistance  $r_e$  corresponds physically with the forward biased emitter barrier resistance which, according to the diode equation, is  $26/I_e$  ohms, where  $I_e$  is the DC bias current in milliamperes. The base resistance  $r_b$  represents the ohmic resistance of the thin base semi-conductor through which the base current flows. The collector resistance  $r_c$  represents the "back" resistance of the reverse biased collector junction. The junction capacitance may be shown for higher frequencies in shunt with  $r_c$ . A Thevenin alternate circuit is to substitute a voltage generator in the output branch, the voltage of which is  $r_m \cdot i_1$  where  $r_m = a \cdot r_c$ . This is not desirable for the reason that the analogy with the physical action of the transistor is lost.

As none of the T parameters are to be found by direct measurement, they must be obtained by conversion from one of the systems of parameters. For example, by comparison of the T and H circuits of figures 4 and 28, we can obtain the h-parameters in terms of the T-parameters by solving the T circuit for the desired h-parameter under the



test condition for the particular h-parameter. The results are given in the Conversion Table. The T-parameters for a given set of h-parameters may be obtained by solution of the relations already obtained.

### CONVERSION TABLE \*

$$z_{11} = \left. \frac{v_1}{i_1} \right|_{i_2} = h_{11} - \frac{h_{12} h_{21}}{h_{22}}$$

$$y_{11} = \left. \frac{i_1}{v_1} \right|_{v_2} = h_{11}$$

$$z_{21} = \left. \frac{v_2}{i_1} \right|_{i_2} = -\frac{h_{21}}{h_{22}}$$

$$y_{12} = \left. \frac{i_1}{v_2} \right|_{v_1} = -\frac{h_{12}}{h_{11}}$$

$$z_{12} = \left. \frac{v_1}{i_2} \right|_{i_1} = \frac{h_{12}}{h_{22}}$$

$$y_{21} = \left. \frac{i_2}{v_1} \right|_{v_2} = \frac{h_{21}}{h_{11}}$$

$$z_{22} = \left. \frac{v_2}{i_2} \right|_{i_1} = \frac{1}{h_{22}}$$

$$y_{22} = \left. \frac{i_2}{v_2} \right|_{v_1} = h_{22} - \frac{h_{21} h_{12}}{h_{11}}$$

$$h_{22} = \frac{1}{r_2 + r_3}$$

$$r_2 = \frac{h_{12}}{h_{22}}$$

$$h_{12} = r_2 h_{22}$$

$$r_3 = \frac{1 - h_{12}}{h_{22}}$$

$$h_{21} = a(1 - h_{12}) - h_{12}$$

$$a = \frac{h_{21} + h_{12}}{1 - h_{12}}$$

$$h_{11} = r_1 + \frac{(1 + a)}{\left(\frac{1}{r_2} + \frac{1}{r_3}\right)}$$

$$r_1 = h_{11} - \frac{h_{12}(h_{21} + 1)}{h_{22}}$$

\* See also the Parameter Conversion Relations. (p 22)







PARAMETERS AND PERFORMANCE  
of a  
2N45 TRANSISTOR

$$I_e = 1.0 \text{ ma}$$

$$V_{cb} = -5.0 \text{ v}$$

|                             | Common Base          | Common Emitter         | Common Collector  |
|-----------------------------|----------------------|------------------------|-------------------|
| $h_{21}$                    | $-\alpha = - .92$    | $\beta = 11.5$         | $-\gamma = -12.5$ |
| $r_{22} = \frac{1}{h_{22}}$ | 1 m                  | 80 k                   | 80 k              |
| $r_o$                       | 148 k                | 148 k                  | 40                |
| $r_i = h_{11}$              | 40                   | 500                    | 500               |
| $r_{11}$                    | 270                  | 270                    | 1 m               |
| $h_{12}$                    | $2.5 \times 10^{-4}$ | $+ 2.5 \times 10^{-4}$ | 1                 |
| $r_e$                       | 20                   |                        |                   |
| $r_b$                       | 250                  |                        |                   |
| $r_c$                       | 1 m                  |                        |                   |
| $a$                         | $- .92$              |                        |                   |

Note: it is only a happenstance that  $h_{12}$  has the same value. The same is true for  $r_{11}$ .



## PERFORMANCE CURVES

The current gain in the three configurations is calculated for a 2N45 transistor by equation 48 and plotted in figure 29. By equations 63 and 64 it is seen that the ratio of the current gain curve of the common emitter to that of the common collector is  $\alpha$ . The value of  $R_L$  which will halve the current gain  $h_{21}$ , is the same for both circuits, and equals  $r_{22}$ . For the common base circuit, it is  $\beta + 1$  larger. It is to be noted that for most practical loading values, the current gain is practically equal to the short circuit current transfer ratio.

The voltage gains are calculated by equation 50 and plotted in figure 30. If equation 50 is written

$$A_v = - \frac{h_{21} r_o}{r_i} \cdot \frac{1}{1 + \frac{r_o}{R_L}} \quad (71)$$

The first term is the open circuit gain. To halve the open circuit gain requires  $R_L = r_o$ . It may be shown by equations 63, 65 and 66 that the voltage gains of the common base and common emitter circuits are exactly the same except for reversal of phase. The common collector circuit, by 63, 65 and 66, has an open circuit gain of unity. Since  $r_{oc} = r_{ib}$ , which is a very low value of resistance, the common collector voltage gain holds up close to unity for low values of load resistance.

Power gains, as calculated by the product of current and voltage gains, are shown in figure 31.



Figure 32 shows how the input resistance  $R_1$  varies as a function of  $R_L$ . Each curve varies between its limits of  $r_i$  and  $r_{11}$ . The internal feedback is responsible for  $R_1$  not always having the value  $r_i$ .

Figure 33 again shows  $R_1$  vs.  $R_L$ . In addition is plotted output resistance  $R_2$  as a function of source resistance  $R_s$ . Due to the opposite kind of internal feedback between the common base and common emitter circuits, the  $R_2$  curves spread apart from the value of  $r_o$ , as  $R_s$  is increased. The feature of plotting as in figure 33, is that the intersections of the  $R_1$  and  $R_2$  curves of a given circuit show the condition of simultaneously matching at both input and output. This is the condition of maximum transistor power gain (eq. 43) or optimum transducer gain (eq. 47). For the 2N45 transistor in the common base circuit this occurs for  $R_s = R_1 = 100$  ohms, and for  $R_L = R_2 = 380$  k. For the common emitter circuit,  $R_s = R_1 = 370$  ohms, and  $R_L = R_2 = 110$  k. For the common collector circuit a very broad intersection results, and the power gain is nearly independent of  $R_L$  between 400 ohms and 10 k as shown in figure 31. The source resistance for an input match may be between 5 k and 100 k, increasing as  $R_L$  is changed from 400 ohms to 10 k.





Problem 1. Sketch the equivalent circuit inside a transistor in the common emitter circuit similar to figure 5. Write equations like 11 and 12.

Problem 2. Derive equation 37.

Problem 3. Show how the output resistance (input AC shorted)  $r_o$  may be obtained graphically by means of the collector and emitter characteristic curves such as figures 7 and 8. Will  $r_o$  be greater or less than  $r_{22}$  for the common base circuit as shown by your graphical construction? Compare the values of  $r_o$  and  $r_{22}$  in the light of the effect which series negative voltage feedback has on output impedance in general.

Problem 4. In which of the input resistance parameters is there the effect of internal feedback? As a result of the negative feedback of the common base circuit, and the positive feedback of the common emitter circuit, explain whether  $r_{11b}$  should be smaller or larger than  $r_{ib}$ , and the same for  $r_{11e}$  and  $r_{ie}$ .

Problem 5. Explain which output resistance parameter should be the larger for the common emitter circuit on the basis of positive feedback. Compare with problem 3.



Problem 6. a. What are the collector static characteristics like in the common collector configuration? Sketch what you would expect the static characteristics to be for this circuit.

b. As a result of feedback compare the relative magnitudes of  $r_{ic}$  and  $r_{llc}$ .

c. Repeat for  $r_{oc}$  and  $r_{22c}$ .

Problem 7. For the common collector circuit, the combined circuit and internal feedback must yield a value of  $h_{12c}$  slightly different from unity. Is it greater or less than unity? Derive an equation for it in terms of  $r_{ib}$  and  $r_{22e}$ . Is the internal feedback positive or negative?

Problem 8. Obtain equation 62 from equations 40, 55, 58 and 59.

Problem 9. Derive equation 68.

Problem 10. Show that -

$$r_{22b} \cdot r_{ib} = r_{ob} \cdot r_{llb} = r_{oc} \cdot r_{llc} = r_{22c} \cdot r_{ic} = r_{22e} \cdot r_{ie} = r_{oe} \cdot r_{lle}$$

Problem 11. Follow the method outlined under "Parameter Conversion Relations" to obtain all four y-parameters in terms of h-parameters. Check with the "Conversion Table".



Problem 12. Repeat #11 for the "z" or "a" parameters. Add the "a" to the Conversion Table.

Problem 13. Convert the h-parameters to those of any other system, such as the z-parameters by the method outlined for the T circuit.

Problem 14. a. Obtain the h-parameters in terms of the T parameters.  
b. Solve these relations explicitly for the T parameters.

Problem 15. Obtain the T parameters from the z parameters.

Problem 16. For some given transistor under given conditions of bias, make a table of numerical values of the parameters listed in the Parameter Conversion Relations Table, for all three configurations.

Problem 17. Make a similar table as above for the equivalent T parameters.

Problem 18. Review problem 3 of the previous chapter by comparing the results with those of the exact method.

Problem 19. Two transistors are to be considered in a transformer coupled audio amplifier stage. The transistors have the following data furnished by the manufacturer for the common base connection:





2N43

$$h_{22} = 1 \text{ meg}$$

$$h_{21} = -.98$$

$$h_{11} = 40 \text{ ohms}$$

$$h_{12} = 4 \times 10^{-4}$$

2N64

$$\beta = 45 \text{ (com. em.)}$$

$$r_1 = r_e = 25 \text{ ohms}$$

$$r_2 = r_b = 700 \text{ ohms}$$

$$r_3 = r_c = 2 \text{ meg.}$$

- a. Calculate the values of load impedance which the output transformer must present to each of the transistors in the common emitter connection to obtain an impedance match, if the input transformer in each case provides input matching.
- b. Which transistor will give a gain of 40 db for the lower value of turns ratio of the output transformer? An output transformer can scarcely be made to have more than 30 k input impedance.
- c. Discuss the feasibility of the other transistor circuits.
- d. If the bias conditions are  $I_e = 1.0 \text{ ma.}$  and  $V_c = 6 \text{ volts}$ , show by load line sketches on the collector characteristics that it is collector bias voltage which limits maximum power output. For which transistor, loaded to give 40 db of gain, will the maximum undistorted power output be the greater?



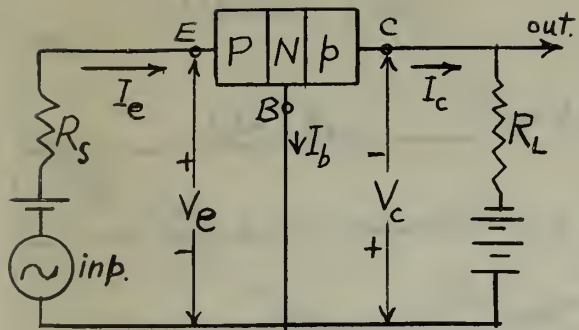


Fig. 1

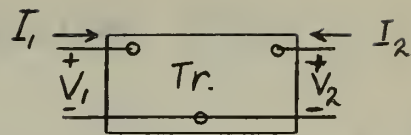


Fig. 2

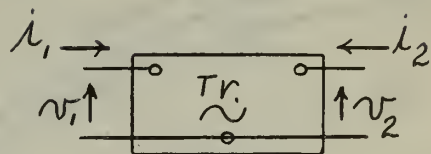


Fig. 3

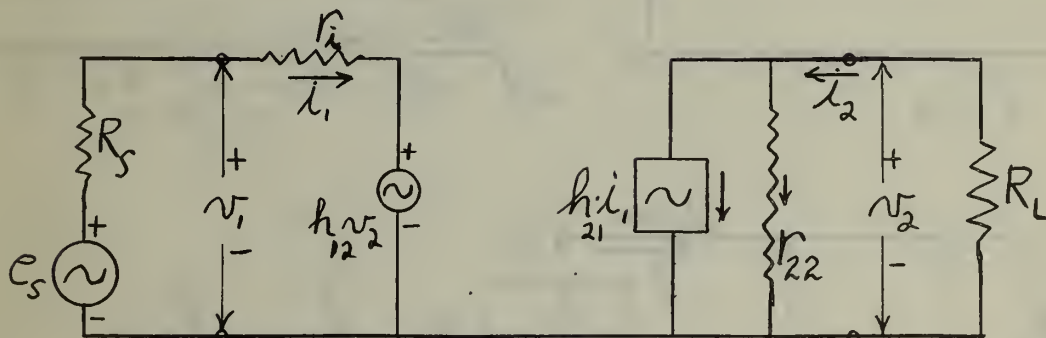


Fig. 4

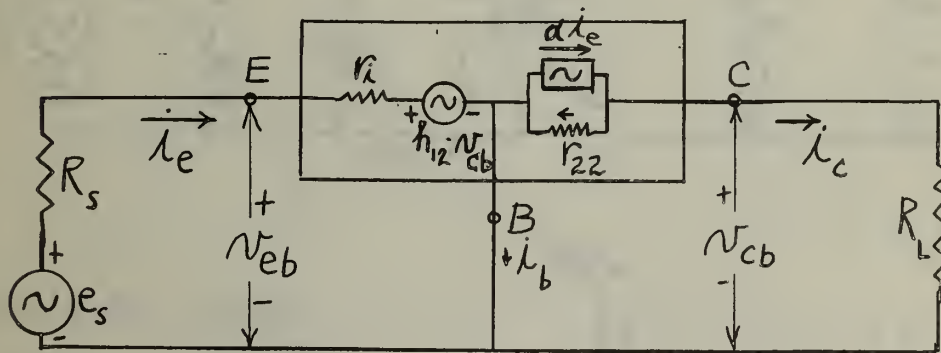


Fig. 5

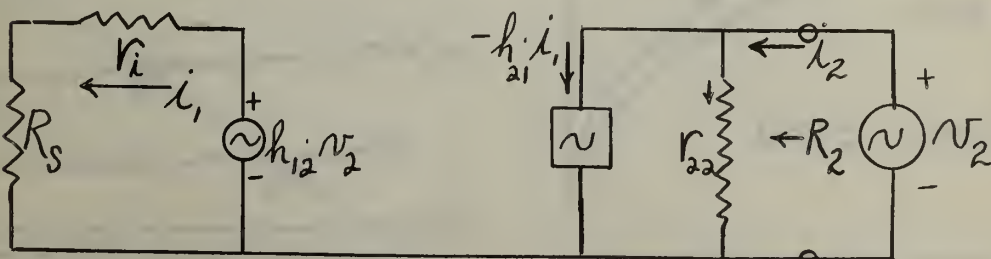
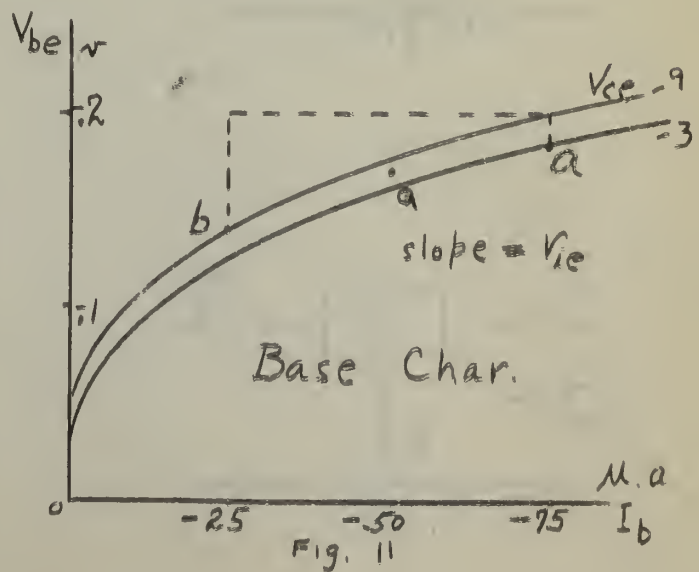
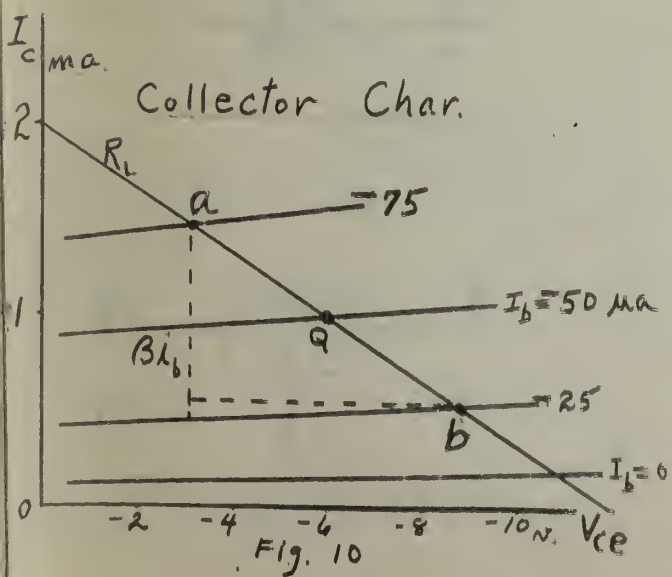
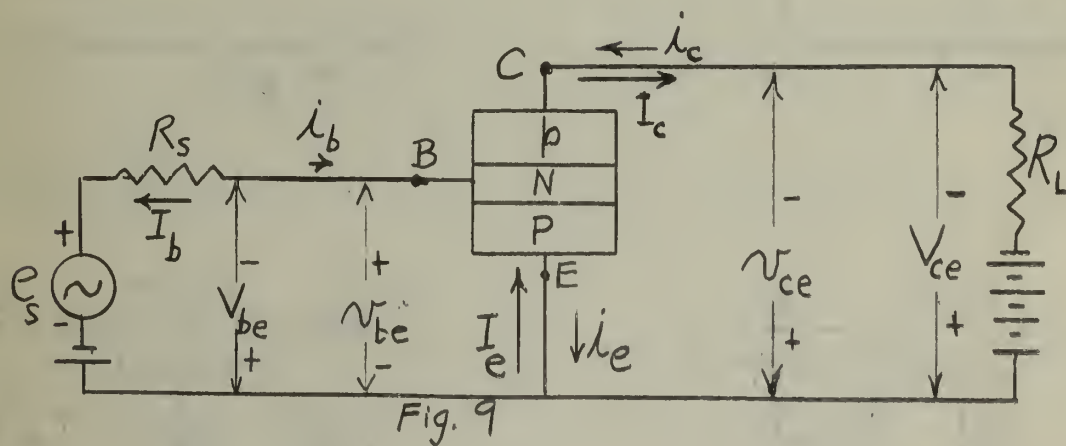
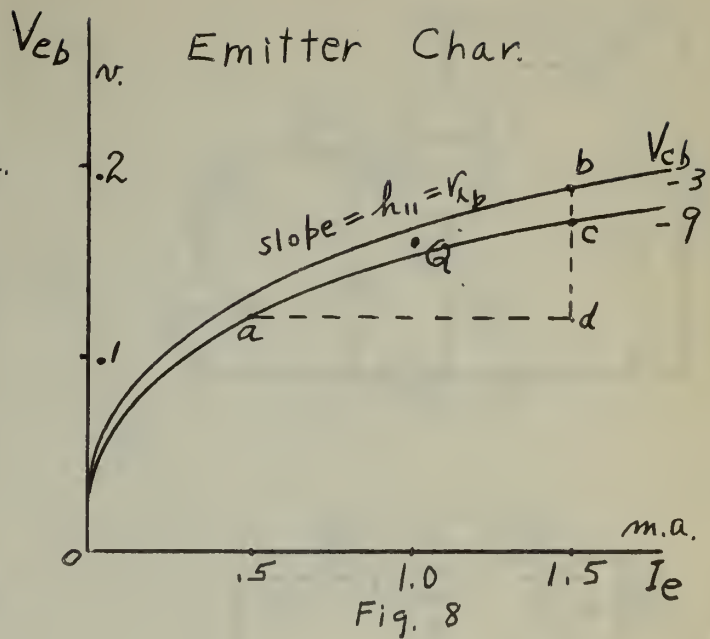
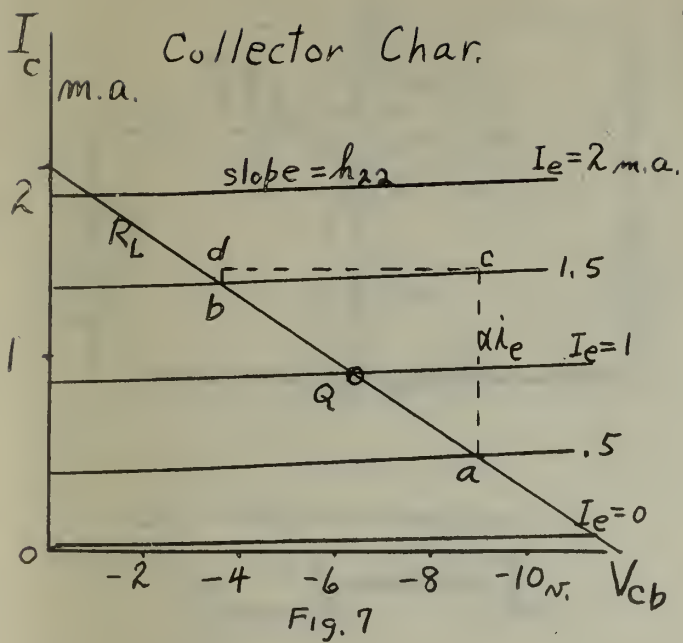


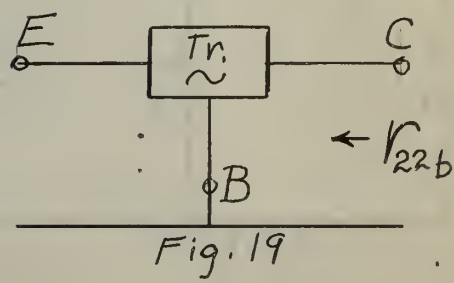
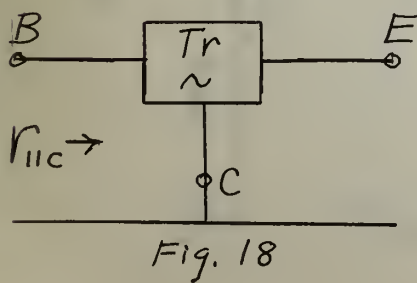
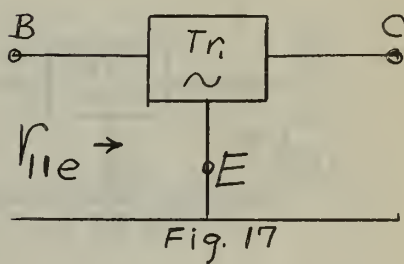
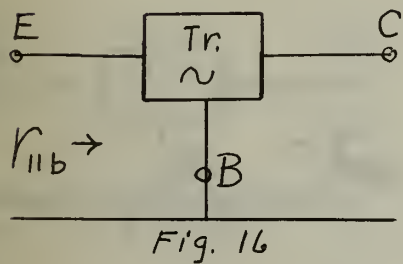
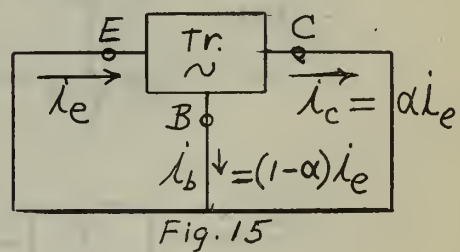
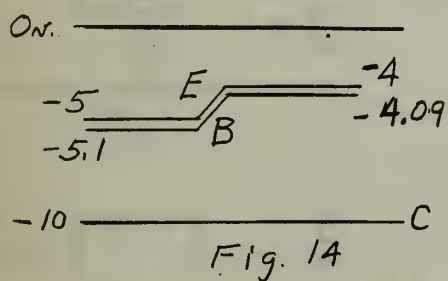
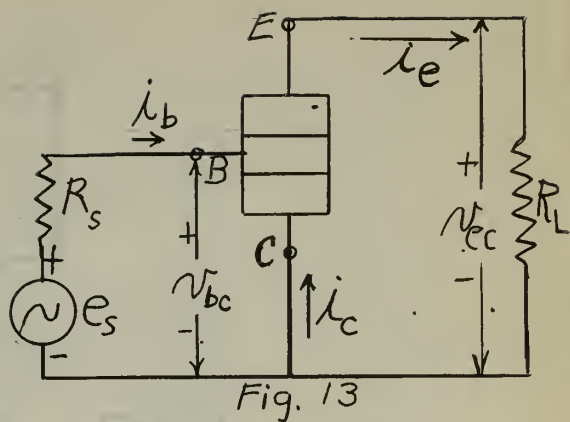
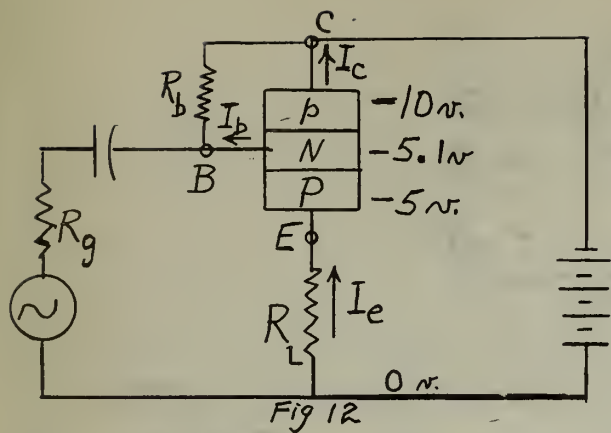
Fig. 6













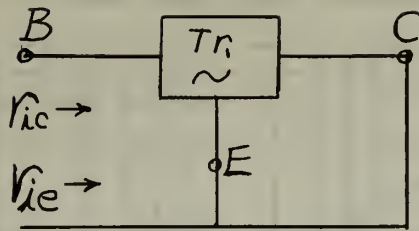


Fig. 20

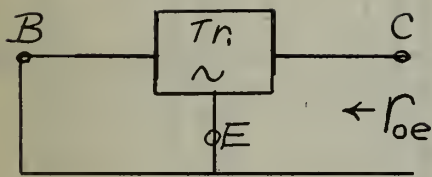


Fig. 21

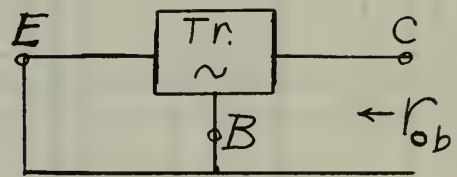


Fig. 22

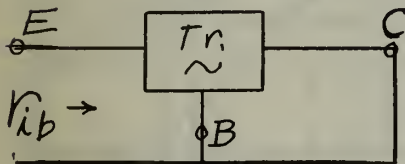


Fig. 23

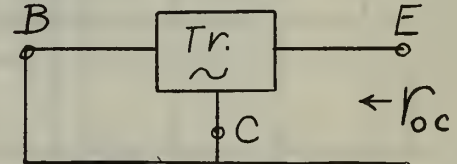


Fig. 24

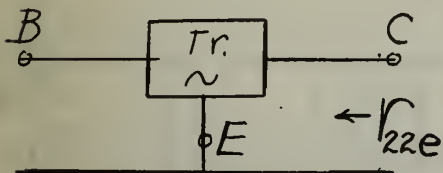


Fig. 25

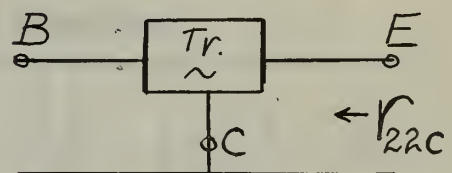


Fig. 26

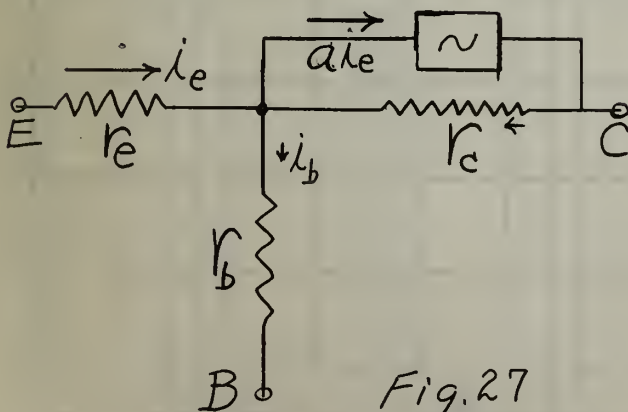


Fig. 27

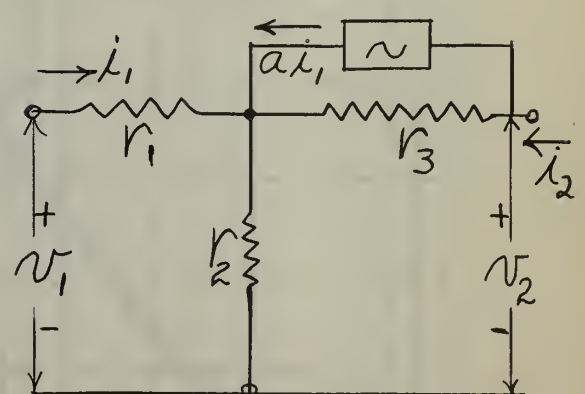


Fig. 28



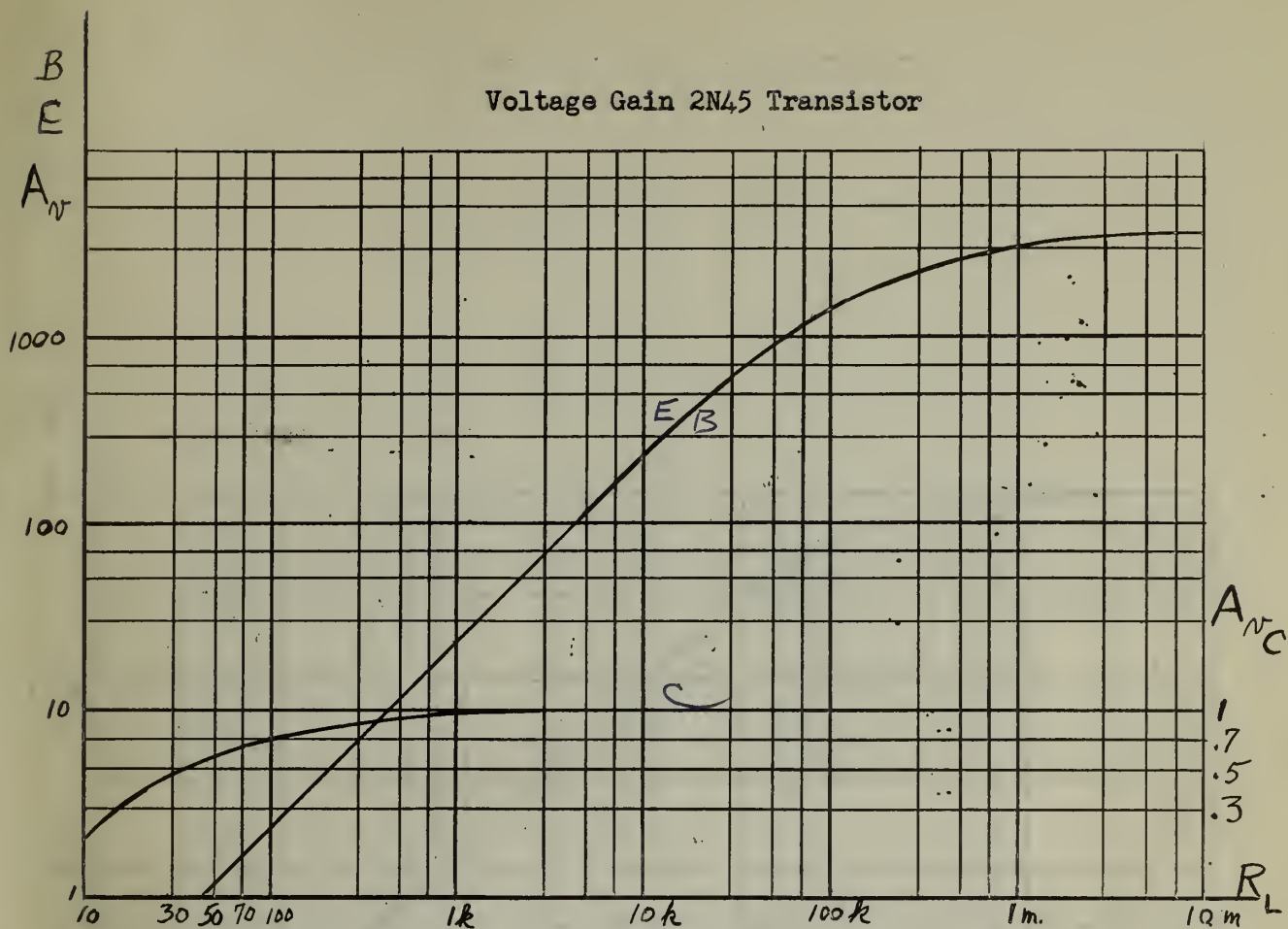


Figure 30

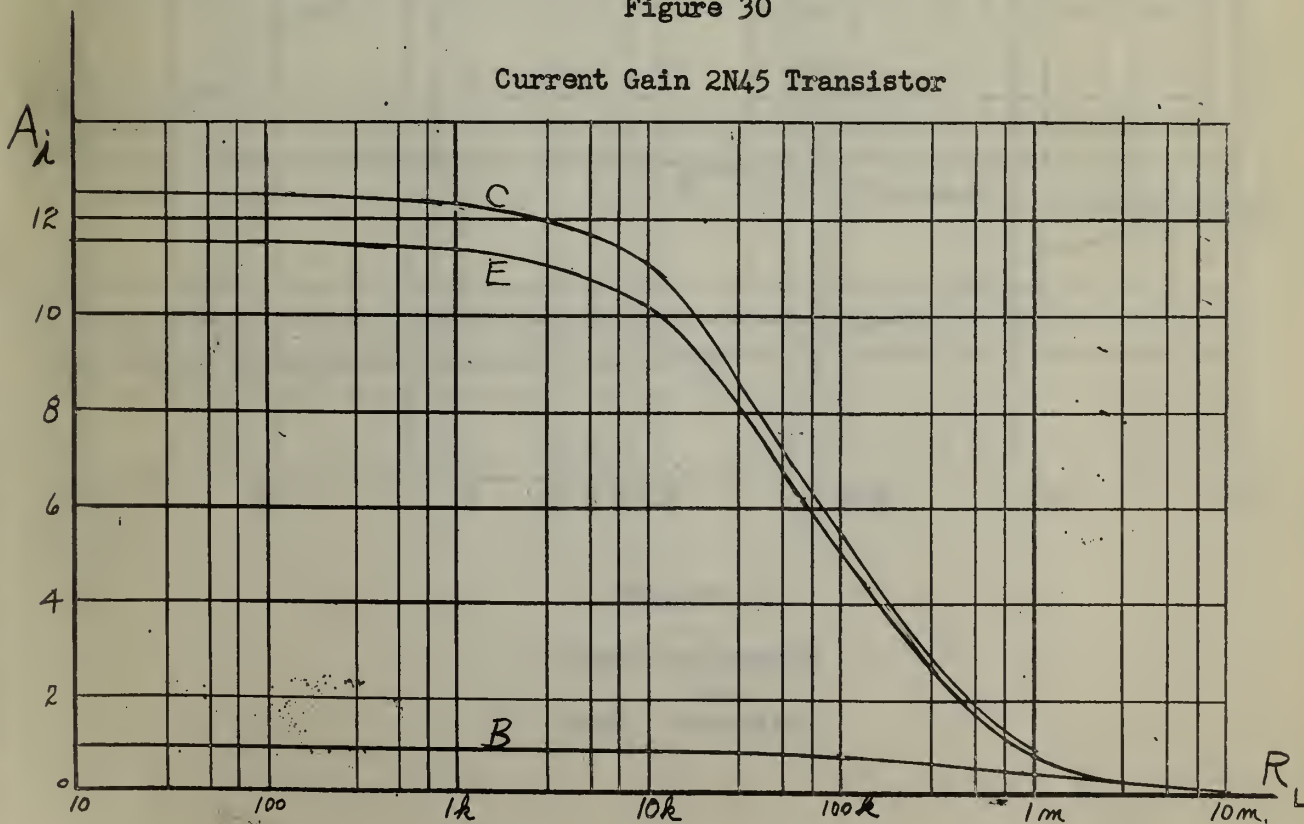


Figure 29







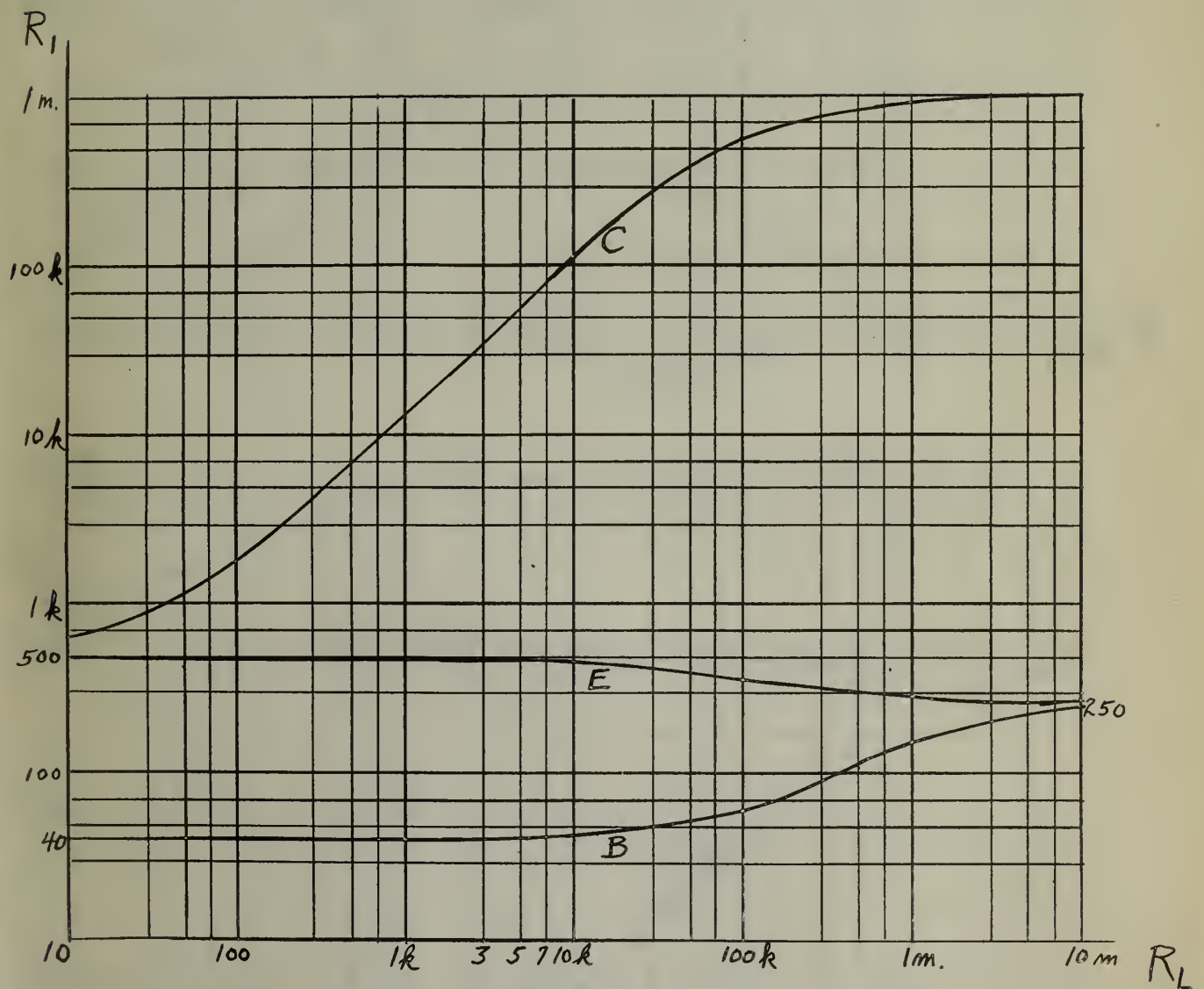
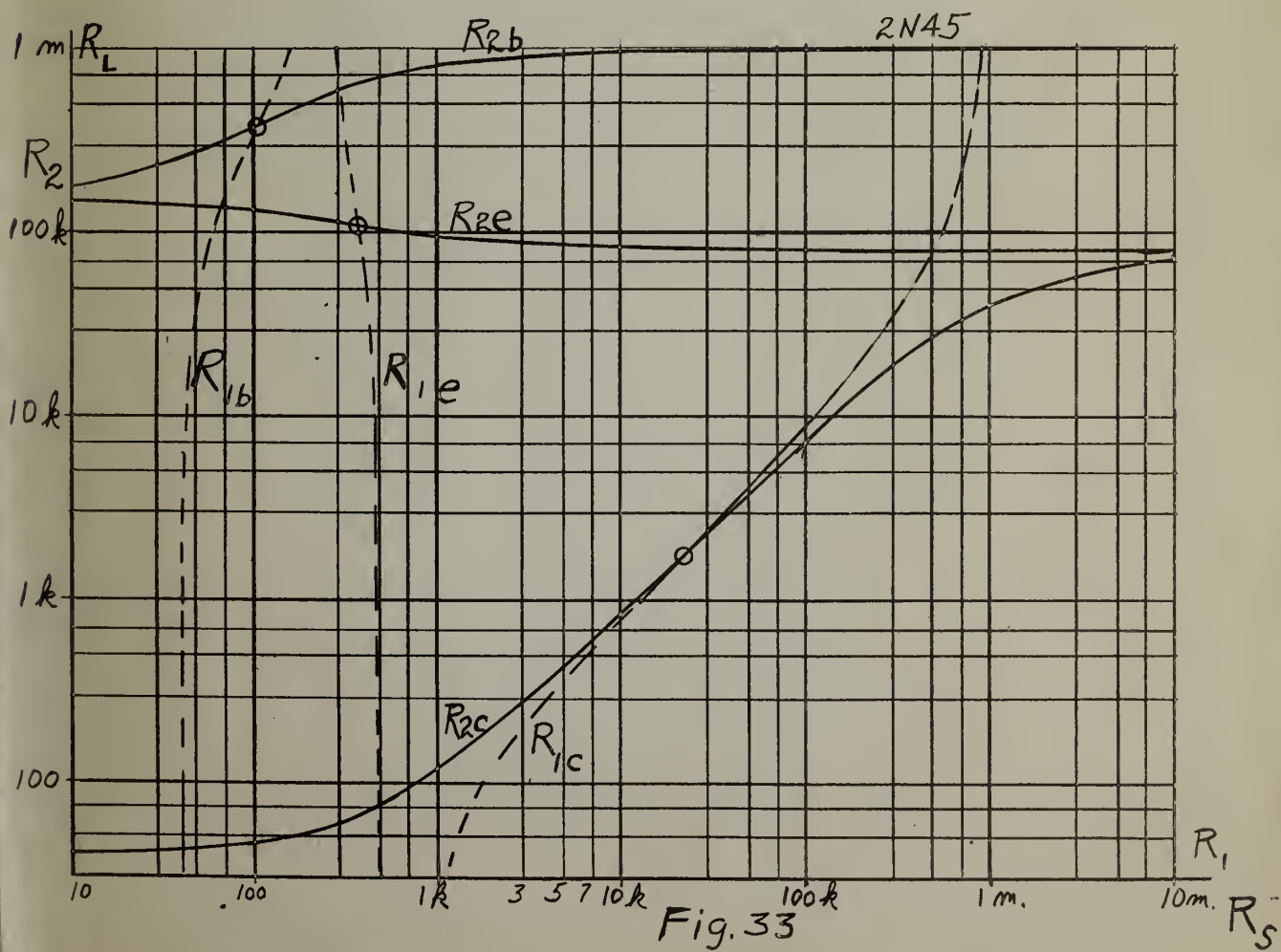
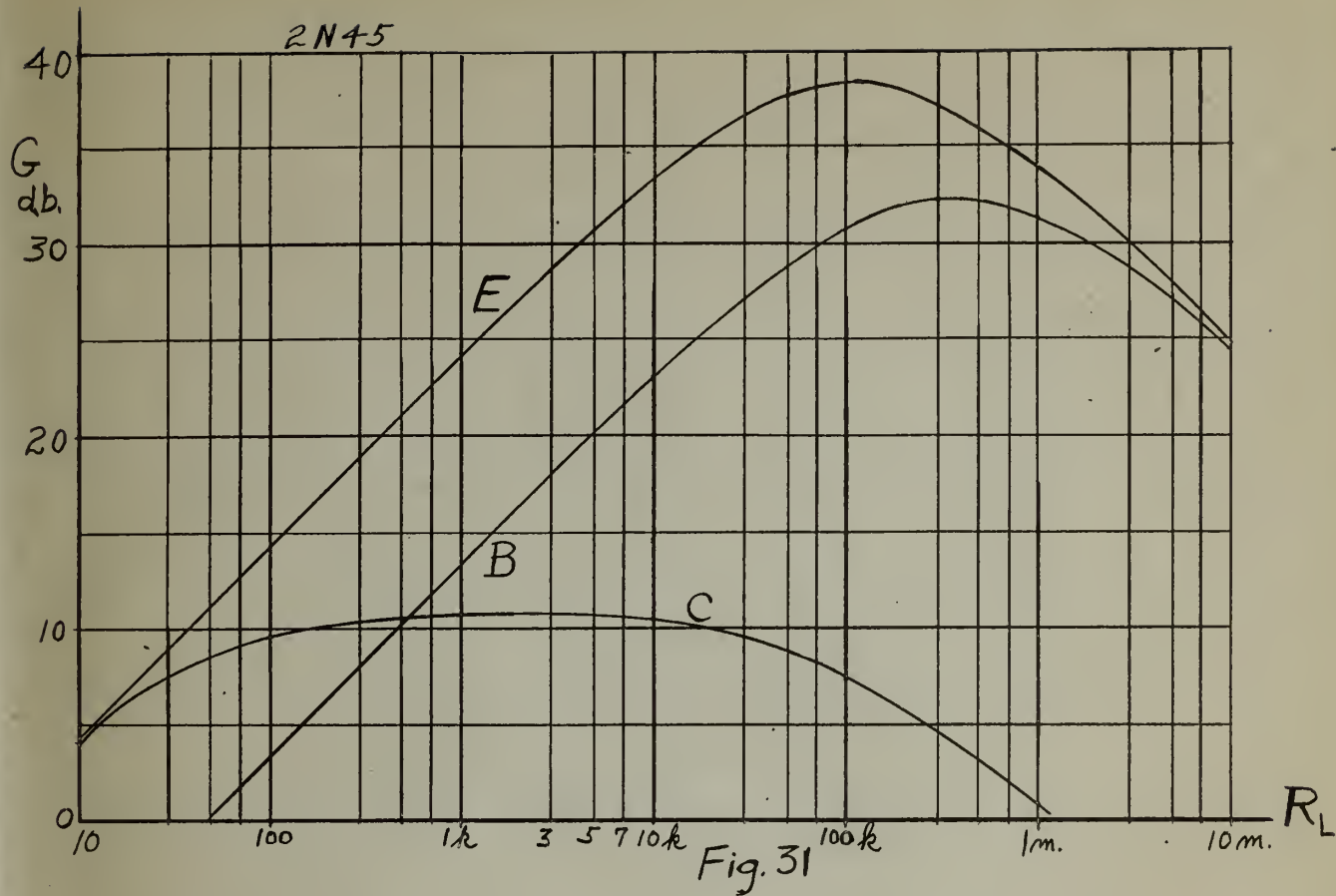


Figure 32

Input Resistance  
of a  
2N45 Transistor













AP 264  
27 JAN 66  
31 OCT 66  
2 JUL 70  
22 APR 71  
17 OCT 77  
17 OCT 77  
12891 DUP  
14819  
14951  
18776  
20633  
22064

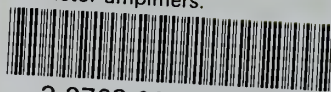
TA7  
.U6  
no.16  
Bauer  
Transistor amplifiers.  
37013

AP 264  
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31 OCT 66  
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22 APR 71  
17 OCT 77  
12891 DUP  
14819  
14951  
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22064

TA7  
.U6  
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Bauer  
Transistor amplifiers.  
37013

genTA 7.U6 no. 16

Transistor amplifiers.



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